Homework #11

Due in class on Wednesday, May 6th, 2015

Reading: Spin network dissertation chapter, sections 3.1, 3.2, 3.3.2, and 3.4.1. Griffiths pp187-188 and section 5.1.

1. Use the spin network techniques that we have been learning to evaluate the number associated with the trefoil knot:

Figure 1: The trefoil knot spin network.

Do this in two different ways: (i) First use the reduction rule to simplify the diagram down to one that you already know the value for. [Note, you don't have to keep using the reduction rule and you may find it easier at points to simply treat the diagram as a string and simplify it that way.] (ii) Now translate the diagram into an algebraic expression involving only ϵ's and δ's and calculate it directly.

2. Using the expression you derived in Griffith's problem 4.56,

$$U = e^{i\vec{\sigma}\cdot\hat{n}\varphi/2} = \cos(\varphi/2) + i(\hat{n}\cdot\vec{\sigma})\sin(\varphi/2),$$

show that spinor rotations are always of determinant one. That is, $\det U = 1$ for all $\hat{n}$.

3. Prove that you can write the determinant from the last problem as

$$\det U = \frac{1}{2}\epsilon^{AB}\epsilon_{CD}U_A^C U_B^D.$$

4. Combining the results of problems 2. and 3. show that

$$U_A^C U_B^D \epsilon_{CD} = \epsilon_{AB}.$$

This means that under the action of spinor rotations the ϵ tensor is invariant! As you know invariants are a wonderful way to understand both mathematics and physics and so this is quite a useful fact.

5. Griffiths Problem 4.35, p189.
6. Griffiths Problem 4.36, p189.