Homework #2

Due in class on Wednesday, February 11th, 2015

Reading: Griffiths Chap. 1, sections 1.4-6. Class notes.

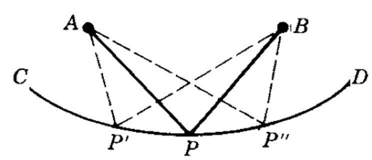
1. In class we used Fermat's principle to derive Snell's law for a ray of light going from air into glass. Instead assume that a water proof flashlight is placed a depth h below the surface of a smooth lake. The water has index of refraction n_w and the air above has index of refraction n_a.

(a) Use Fermat's principle to derive the critical angle θ_c (with respect to the normal of the surface) at which you should shine the flashlight to have the resulting ray skim the surface and hit a detector floating on the lake.

(b) Look up reasonable values for n_w and n_a and calculate the value of θ_c.

(c) If you further tilt the flashlight to $\theta > \theta_c$ what does the ray do? Again apply Fermat's principle to relate the angle of the ray before meeting the surface to that after. (The figure at right illustrates this lovely phenomenon.)

2. Show that the actual path of a light ray is not necessarily one of minimum time. Hint: In the diagram, A is a source of light; CD is a cross section of a reflecting surface, and B is a point to which a light ray is to be reflected. APB is to be the actual path and $AP'B$, $AP''B$ represent varied paths. Then show that the varied paths:



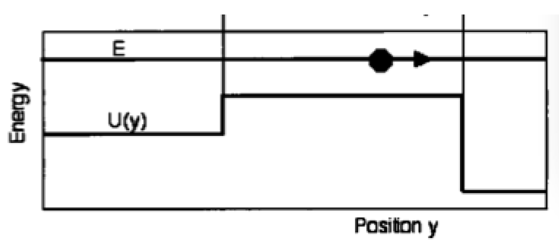
(a) Are the same length as the actual path if CD is an ellipse with A and B as foci.

(b) Are longer than the actual path if CD is a line tangent at P to the ellipse in (a).

(c) Are shorter than the actual path if CD is an arc of a curve tangent to the ellipse at P and lying inside it. Note that in this case the time is a maximum!

(d) Are longer on one side and shorter on the other if CD crosses the ellipse at P but is tangent to it (that is, CD has a point of inflection at P).

3. Suppose a bead moves along a long straight wire and is subject to the varying potential depicted at right.



(a) Supposing the bead's energy is conserved, draw a qualitatively accurate plot of the bead's velocity as a function of its position over the whole of the pictured y range.

(b) Redraw this plot for a bead that briefly reverses direction at the position where the black bead is depicted, then reverses direction again and completes the journey to the end of the y range. Note

that this need not violate energy conservation.
(c) Argue that (a) is the physical trajectory and (b) is not. [Hint: What does Hamilton's principle have to say about these two trajectories?]

4. Griffiths Problem 1.5.
5. Griffiths Problem 1.7.
6. Griffiths Problem 1.9.
7. Griffiths Problem 1.14.
8. Griffiths Problem 1.15.
9. Griffiths Problem 1.16.