Homework #3

Due in class on Wednesday, February 18th, 2015

Reading: Griffiths Chap. 3, sections 3.1-3. Appendix A.1-A.2. Class notes.

1. We have discussed Fourier series as an example of viewing functions as vector spaces and for illustrating bases of functions. However, thus far I haven't shown you the details that justify some of these claims. Let's go through some of these here. First let's explore orthonormality. (a) By hand show that one of the following equations holds:

$$\begin{aligned} \int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx &= \delta_{mn} \\ \int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx &= \delta_{mn} \\ \int_{-\pi}^{\pi} \cos(mx)\sin(nx)dx &= 0 \end{aligned} \tag{1}$$

[Hint: You will probably want to use a trigonometric identity to simplify the integrand first and reduce this to a sum of integral you know how to do.]
(b) Confirm that the other two equalities are true using a computer algebra package; you can simply mark this part as done.
(c) Now, assume that we have a function $f(x)$ that is periodic with period 2π and that it can be expanded as

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx). \tag{2}$$

(The fact that any such periodic function can be expanded this way is called **completeness** and we won't prove it here; it's a nice thing to tackle in a math class.) If I am given the function $f(x)$ how do I find a_0? Similarly prove one of the following formulas by hand:

$$\begin{aligned} a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx. \end{aligned} \tag{3}$$

(d) Find the Fourier series, that is the a_0, a_n, and b_n, for the periodic function

$$f(x) = \frac{x}{\pi}, \qquad -\pi < x < \pi. \tag{4}$$

(e) Plot $f(x)$, as well as $a_0 + a_1\cos(x) + b_1\sin(x)$, $a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x)$ and so on including terms up to $n = 4$.

2. Griffiths Problem 3.3.
3. Griffiths Problem 3.5 [skip part (b)].
4. Griffiths Appendix A, Problem A.2.
5. Griffiths Appendix A, Problem A.4.

6. Use the Gram-Schmidt procedure (see Problem A.4) to orthonormalize the functions $1, x, x^2$, and x^3 on the interval $-1 \le x \le 1$—they are (apart from the normalization) the **Legendre polynomials**, see Table 4.1 of Griffiths.