Liam's guest Lecture on Angular Momentum

I Define L

II Commutation properties

<u>Classically</u>

I. $\vec{L} = \vec{r} \times \vec{p}$ $\quad \vec{L} = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{pmatrix}$

So we find

order doesn't matter

$$L_x = y p_z - z p_y \longrightarrow \hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$$

because $\hat{y}\hat{p}_z = \hat{p}_z \hat{y}$.

At this point he'll drop the hat notation on operators to save time. So the other operators are

$$L_y = z p_x - x p_z$$

$$L_z = x p_y - y p_x$$

Do these operators commute?

$$[L_x, L_y] = [yP_z - zP_y, zP_x - xP_z]$$

$$= [yP_z, zP_x] - [yP_z, xP_z] - [zP_y, zP_x] + [zP_y, xP_z]$$

$$= yP_x[P_z, z] - yx\cancel{[P_z, P_z]}^{0} - P_yP_x\cancel{[z, z]}^{0} + xP_y[z, P_z]$$

$$= -yP_x[z, P_z] + xP_y[z, P_z]$$

$$= i\hbar(xP_y - yP_x) = i\hbar L_z \quad \checkmark$$

Then
$$\boxed{[L_x, L_y] = i\hbar L_z}$$

This means that there are not simultaneous e-states of L_x and L_y (or any pair of ang. momentum directions).

We have,

$$\sigma_{L_x}\sigma_{L_y} \geq \left|\left(\frac{1}{2i}\langle i\hbar L_z\rangle\right)\right|$$

$$= \frac{\hbar}{2}|\langle L_z\rangle|$$

Despite the fact that the components don't commute, we can consider

$$[L^2, L_x] \quad \text{where } L^2 = L_x^2 + L_y^2 + L_z^2$$

then

$$[L^2, L_x] = \cancelto{0}{[L_x^2, L_x]} + [L_y^2, L_x] + [L_z^2, L_x]$$

<u>Fact</u>: $[A^2, B] = A[A,B] + [A,B]A$

So

$$[L^2, L_x] = L_y[L_y, L_x] + [L_y, L_x]L_y + L_z[L_z, L_x] + [L_z, L_x]L_z$$

$$= L_y(-i\hbar L_z) + (-i\hbar L_z)L_y + L_z(i\hbar L_y) - i\hbar L_y L_z = 0 \checkmark$$

Then

$$[L^2, \vec{L}] = 0$$

Comments: Nice that there were no skipped steps