Zechen's guest lecture on Bell inequalities May 11th, 2015 P1/6

Einstein: "Do you really believe that the moon only exists when you look at it?"

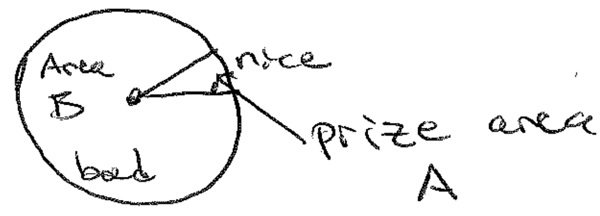
Big question: * Is the moon there when no one looks at it?

RA: Realist answer: yes! EPR propose the challenge of their paradox. 1930's

OA: Orthodox answer: No!

~~RA~~ Why is the RA awesome?
hidden ~~var~~ variable ⇝ classical statistics

Roulette in Las Vegas




$t=0$ start

$t=10\,sec$ stop

$$P(\text{prize}) = \frac{A}{A+B}$$

What's the variable behind this result? Ang. vel. $\omega$.

If you know $\omega$ (and where you start) exactly, then you know if you'll get a prize.

We can describe $\omega$ by a distribution too $f(\omega)$

If there were a hidden variable we could likely understand the Q.M. results in a similar manner!

<u>OA</u>: John Stewart Bell 1964

Ask an ensemble of systems 3 questions (yes or no type)

Say $\underline{a}$, $\underline{b}$, $\underline{c}$

Yes $A$ $B$ $C$

No $\bar{A}$ $\bar{B}$ $\bar{C}$

Denote the # of systems that answer A and not B is $N(A,\bar{B})$. Consider

$$N(A,\bar{B}) + N(B,\bar{C}) \geq N(A,\bar{C})$$

we will briefly prove this inequality.

a. Do you like salads in Kline?
b. " hamburgers " ?
c. " pizza " ?

Surveying the room he finds

$$N(A,\bar{B}) + N(B,\bar{C}) \geq N(A,\bar{C})$$

$$4 + 1 > 2 \quad !$$

It worked. Proof sketch

$$\begin{array}{ccccc} N(A,\bar{B}) & + & N(B,\bar{C}) & \geq & N(A,\bar{C}) \\ \| & & \| & & \| \\ N(A,\bar{B},C) & & \cancel{N(A,B,\bar{C})} & & \cancel{N(A,B,\bar{C})} \\ + \cancel{N(A,\bar{B},\bar{C})} & & + & & + \\ & & N(\bar{A},B,\bar{C}) & & \cancel{N(A,\bar{B},\bar{C})} \end{array}$$

$$\Rightarrow N(A,\bar{B},C) + N(\bar{A},B,\bar{C}) \geq 0$$

Running the argument backwards gives a proof. Big deal? What about the EPR experiment?

Let's run a, b, c on the EPR experiment.

a. Is the spinor up or down in the z-direction? $\uparrow_{0^\circ}$ $\downarrow_{0^\circ}$

b. " up or down at angle $\theta$ in the $yz$ plane?

c. " up or down at angle $2\theta$ in $yz$ plane?

1st exp. $P(\uparrow_0, \downarrow_\theta)$

yes is up
no is down.

2nd exp. $P(\uparrow_\theta, \downarrow_{2\theta})$

3rd exp. $P(\uparrow_0, \downarrow_{2\theta})$

Recall the state of the pair is $\frac{1}{\sqrt{2}}(\uparrow_0\downarrow_0 - \downarrow_0\uparrow_0)$.

He claims that the $\theta$ measurements are related to a change of basis.

Recall,

$$e^{i(\vec{\sigma}\cdot\vec{n})\phi/2} = \cos\left(\frac{\phi}{2}\right)I + i(\vec{n}\cdot\vec{\sigma})\sin(\phi/2)$$

For $\vec{n} = (n_x, 0, 0)$ we get

$$= \left[\cos(\phi/2)I + i\sigma_x \sin(\phi/2)\right]\uparrow_0 = \uparrow_\theta$$

$$= \begin{pmatrix} \cos\phi/2 & i\sin\phi/2 \\ i\sin\phi/2 & \cos\phi/2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\phi/2 \\ i\sin\phi/2 \end{pmatrix}$$

$$= \cos\phi/2 \uparrow_0 + i\sin\phi/2 \downarrow_0$$

So $P(\uparrow_0 \downarrow_\theta) = P\left(\cos\cancel{\phi}/2 \uparrow_0 \uparrow_0 + i\sin\phi/2 \uparrow_0 \downarrow_0\right)$

$$= \sin^2\frac{\cancel{\theta}}{2}$$

Then everything rotated by $\theta$

$$P(\uparrow_\theta, \downarrow_{2\theta}) = \sin^2\theta/2$$

and

$$P(\uparrow_0, \downarrow_{2\theta}) = \sin^2\theta$$

So is

$$P(\uparrow_0, \downarrow_\theta) + P(\uparrow_\theta, \downarrow_{2\theta}) \geq P(\uparrow_0, \downarrow_{2\theta}) \; ?$$

Well,

$$\sin^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} \geq \sin^2\theta \quad ?$$

Well, for small $\theta$

$$\frac{\theta^2}{4} + \frac{\theta^2}{4} \ngeq \theta^2$$

This is violated.

Can we test it? In 1981 Alain Aspect, performed this experiment and Q.M. violated the Bell inequality.

Buddy[illegible]