Quantum Mechanics

Feb 18th, 2015 P1/2

Lect. 11

I Last time

II ~~Appl.~~ Time independent Schrödinger Eq and stationary states

III Applications: Loren's guest lecture on the infinite square well

- If $\dim \mathcal{H}$ = finite then eigenvectors of $\hat{Q}$ span $\mathcal{H}$

we take as an axiom in continuous case

• Found e-functions of $\hat{p}$ to illustrate continuous spectrum

$$f_p = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$$

Not normalizable, but Dirac

I • Took our first look at observable determinate states

~~...~~ Discrete spectra

- e-values of hermitian $\hat{Q}$ are real
- e-functions for distinct e-values are orthogonal

orthonormal

$$\langle f_{p'} | f_p \rangle = \delta(p - p')$$

They are complete!

Also noticed

$$\lambda = \frac{2\pi}{k} = \frac{2\pi\hbar}{p} = \frac{h}{p}$$

de Broglie!

II Let's return to

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

Let us assume $V(x,t) = V(x)$, that is, no t dependence. Then we can separate variables. Assume

$$\Psi(x,t) = \psi(x)\,\varphi(t)$$

Then

$$i\hbar\,\psi\,\frac{d\varphi}{dt} = -\frac{\hbar^2}{2m}\varphi\frac{d^2\psi}{dx^2} + V\varphi\psi$$

and if we divide through by $\psi\varphi$ we get

$$\underbrace{i\hbar\frac{1}{\varphi}\frac{d\varphi}{dt}}_{\text{only a function of } t} = \underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V}_{\text{only a function of } x}$$

(here's where $V = V(x)$ comes in.)

When can $f(t) = g(x)$?

Only possible if $f = g = \text{const.} \equiv E$. ← call it

But these separable states are only special solutions of the Schrödinger equation. Who cares about 'em?!

Three reasons to care — we'll cover two now:

1. They are stationary states.
2. They have definite total energy.

Then

$$\frac{d\varphi}{dt} = -\frac{i}{\hbar}E\varphi$$

and

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi}$$

We can solve for φ ~~for~~ once and for all

$$\varphi = e^{-\frac{i}{\hbar}Et}$$

↑ we'll put the overall constant in ψ.