Quantum Mechanics

Lect. II

Jan 28th, 2015

I Where we left off...

II Transformations

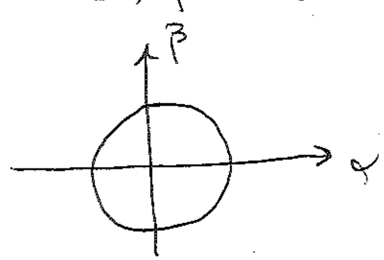
III Interference

IV Physical example
← didn't get to this

We can use the 2-norm if we interpret the squares as the probabilities. For example take (α, β) as variables, we want

$$|\alpha|^2 + |\beta|^2 = 1$$

All such (α, β) form a circle



I Could quantum mechanics be otherwise?

Restricted attention to real numbers for the moment.

Intro'd 1-norm

$$P_1 + \dots + P_N = \sum_i P_i = 1, \quad P_i \in \mathbb{R}_+$$

and 2-norm

$$|\alpha|^2 + |\beta|^2 + \dots + |\omega|^2 = 1, \quad \alpha, \beta, \dots \in \mathbb{R}$$

But then, why not just forget about α and β and only work with $|\alpha|^2$ and $|\beta|^2$?

II Any answers to this?

The difference is striking when you consider transformations!

In probability theory a valid transformation must preserve the 1-norm of your state, e.g. starting with $\begin{pmatrix} p \\ 1-p \end{pmatrix}$, a bit, one valid transformation is

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} 1-p \\ p \end{pmatrix},$$

which is sometimes called a bit flip. Did this result in a valid probabilistic description of the transformed bit? Yes! Because it preserved the 1-norm.

But, if we work with $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ and the 2-norm

$$|\alpha|^2 + |\beta|^2 = 1, \quad \alpha, \beta \in \mathbb{R}$$

what is the most general matrix

$$O = \begin{pmatrix} r & s \\ t & u \end{pmatrix}$$

that preserves this norm?

This time the answer may be quite familiar, one big class

Q: What are the conditions on a completely generic matrix

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

for it to give a valid transformation of a probabilistic bit? These are called stochastic matrices

are the rotations

$$O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

These can be supplemented by the parity

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

to give the full orthogonal group.

An equivalent characterization

(to preserving the 2-norm) is that

$$\tilde{O} = O^{-1}.$$

III This 2-norm bit that we've been studying is called a qubit. Let's denote the two possible measurement outcomes for the qubit

1 (like a "click")

0 (like no click)

Starting with the state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, i.e. $|0\rangle$, we get

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{pmatrix}$$

The resulting amplitudes $1/\sqrt{2}$ and $1/\sqrt{2}$ correspond to a 50-50 probability for the two outcomes. We've "flipped" a coin. But, now let's do it again

Throughout the semester we will use Dirac's notation in which (α, β) is written

$$\alpha|0\rangle + \beta|1\rangle$$

"amplitude" of outcome 0   "amplitude" of outcome 1.

Let's use our new ability to transform outcomes and rotate the amplitudes by 45°.

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Now the outcome is certain! By flipping a flipped coin we get a definite answer. Notice the essential role the minus sign played here! The two norm matters.