Lects 20 & 21

I Last time

II Chen on the delta function well

III Solve delta ~~barrier~~ scattering together

IV Survey

I • Interpreting

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

requires care.

• Showed that this can be interpretted precisely using

$$\Delta E \Delta t \geq \hbar/2$$

with

$$\Delta E \equiv \sigma_H$$

and

$$\Delta t = \frac{\sigma_Q}{\left|\frac{d\langle Q\rangle}{dt}\right|}.$$

II Chen's guest lecture

III For scattering $E>0$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar}\psi = -k^2\psi$$

with

$$k = \frac{\sqrt{2mE}}{\hbar}$$

Solutions:

$$\psi(x) = A e^{ikx} + B e^{-ikx} \quad (x<0)$$

For $x>0$ have same equation, so

$$\psi(x) = F e^{ikx} + G e^{-ikx} \quad (x>0)$$

Continuity at $x=0$ gives

$$A + B = F + G \qquad (1)$$

For the derivatives: 1^{st} $(x<0)$

$$\frac{d\psi}{dx} = ik\left(Ae^{ikx} - Be^{-ikx}\right) \Rightarrow \left.\frac{d\psi}{dx}\right|_{0^-} = ik(A-B)$$

$(x>0)$

$$\frac{d\psi}{dx} = ik\left(Fe^{ikx} - Ge^{-ikx}\right) \Rightarrow \left.\frac{d\psi}{dx}\right|_{0^+} = ik(F-G)$$

So, $\Delta\left(\frac{d\psi}{dx}\right) = ik\left[F-G-A+B\right]$

from chen

$$\stackrel{d}{=} -\frac{2m\alpha}{\hbar^2}\psi(0) = -\frac{2m\alpha}{\hbar^2}(A+B)$$

So $G=0$, Now we can solve for B and F in terms of A

$$B = \frac{i\beta}{1-i\beta}A \qquad F = \frac{1}{1-i\beta}A$$

The relative probability of reflection

$$R = \frac{|B|^3}{|A|^2} = \frac{\beta^2}{1+\beta^2}$$ "reflection coefficient"

Tells you fraction of particles that bounce back.

We learn that

(2)

$$F-G = A(1+2i\beta) - B(1-2i\beta)$$

where $\beta = \frac{m\alpha}{\hbar^2 k}$

A Send wave in $\longrightarrow$

B scattered wave $\longleftarrow$

F transmitted wave $\longrightarrow$

G Send wave in $\xleftarrow{x}$

& not done

Similarly,

$$T = \frac{|F|^3}{|A|^2} = \frac{1}{1+\beta^2}$$

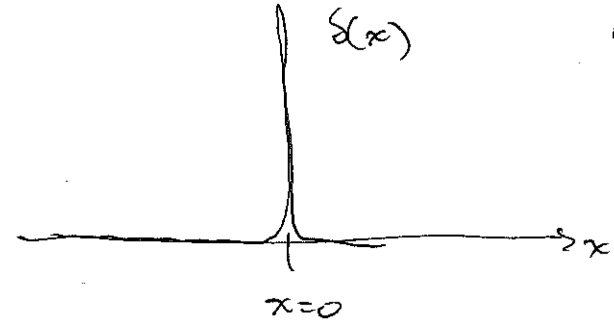
Note,

$R + T = 1$ ✓

In terms of E:

$$R = \frac{1}{1+\left(\frac{2\hbar^2 E}{m\alpha^2}\right)}$$

$$T = \frac{1}{1+\left(\frac{m\alpha^2}{2\hbar^2 E}\right)}$$



How do we interpret these results?

The wavefunctions are not normalizable.

You proceed as before with wave packets — easy idea but difficult in practice and is often implemented on a computer.

This results upon $\alpha \mapsto -\alpha$ (no bound state)

$$R = \frac{1}{1 + \left(\frac{2\hbar^2 E}{m\alpha^2}\right)}$$

$$T = \frac{1}{1 + \left(\frac{m\alpha^2}{2\hbar^2 E}\right)}$$

What if we did a δ-function barrier?

The particle is transmitted and reflected in the same way!

The particle has a non-zero probability of going through an ∞ barrier!!

It doesn't go over — instead it tunnels through makes wonderful electronics and microscopy experiments possible (e.g. Josephson junctions and STM).

Survey:

① How is the pace of the lectures on the whole?
(Written as well as numerical feedback is helpful:

0	5	10
Too slow	just right	too fast

② What topic, if any, is most unclear to you still? Which is clearest? Why for both?

③ Which aspect of class has been most effective for you? Which has been least?

④ What suggestions do you have for improving the class overall?

⑤ For those of you that have had the practice lecture feedback, did you find it discouraging or helpful? For all: How do you feel about having this aspect of the class graded?