Quantum Mechanics
Lect. 23

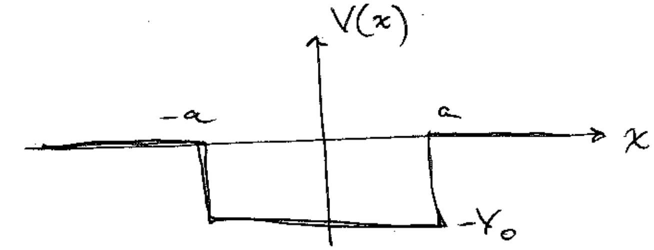
Mar 25th, 2015

I Last time

II Liam's guest lecture introducing angular momentum operators

III Angular Momentum Eigenvalues

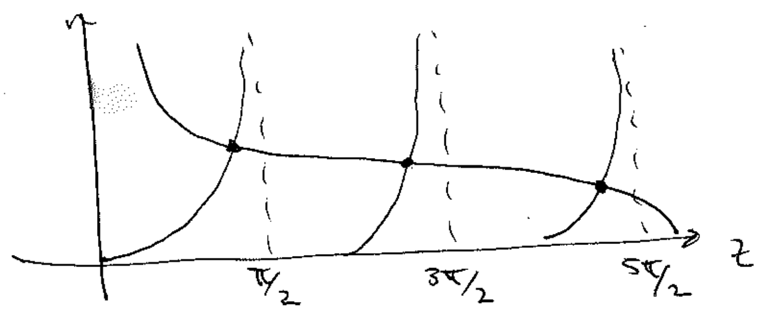
I We solved for the <u>even</u> bound states of the finite square well



We found that the bound state energies were determined by

$$\tan(z) = \sqrt{(z_0/z)^2 - 1}$$

Qualitatively



A limit: <u>wide, deep well</u>

z_0 large $\Rightarrow$ $\tan(z)$ large
$\left(= \frac{a}{\hbar}\sqrt{2mV_0}\right)$

$$\Rightarrow z \approx \frac{n\pi}{2} \equiv z_n$$

$(z \equiv la)$

then

$$z_n = l_n a = \frac{a\sqrt{2m(E_n+V_0)}}{\hbar} = \frac{n\pi}{2}$$

or

$$E_n + V_0 = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$$

The sq. well energies! (shifted by V_0)

II See guest lecture notes

III Summary

$[L_x, L_y] = i\hbar L_z$ and cyclic permutes

$[L^2, L_x] = 0$ and y and z.

$\sigma_{L_x} \sigma_{L_y} \geq \frac{\hbar}{2} |\langle L_z \rangle|$

So, L_z and L^2 are compatible observables. Let's find their eigenvalues:

$$L^2 f = \lambda f \quad \text{and} \quad L_z f = \mu f \quad (*)$$

We'll try to copy what we did for the oscillator.

$$L_\pm \equiv L_x \pm i L_y$$

Then

$$[L_z, L_\pm] = [L_z, L_x] \pm i [L_z, L_y]$$
$$= i\hbar L_y \pm i(-i\hbar L_x)$$

So

$$\boxed{[L_z, L_\pm] = \pm \hbar (L_x \pm i L_y) = \pm \hbar L_\pm}$$

and

$$[L^2, L_\pm] = 0.$$

Claim: If f satisfies eqn.s $(*)$ then so does $L_\pm f$, except with $L_z(L_\pm f) = (\mu \pm \hbar)(L_\pm f)$.

Pf: $L^2(L_\pm f) = L_\pm(L^2 f) = L_\pm(\lambda f) = \lambda(L_\pm f)$ ✓

$$L_z(L_\pm f) = (L_z L_\pm - L_\pm L_z) f + L_\pm L_z f$$
$$= \pm\hbar L_\pm f + \mu L_\pm f$$
$$= \boxed{(\mu \pm \hbar)(L_\pm f)} \checkmark$$

So again L_+ raises the e-value from μ to $\mu + \hbar$ and L_- lowers it to $\mu - \hbar$, but they both leave L^2 alone.