Quantum Mechanics Mar 27th, 2015

Lect. 24

I Last time

II Finding μ and λ.

III ~~Finding angular momentum eigenfunctions~~
(ran out of time)

I. We saw

$$[L_x, L_y] = i\hbar L_z \quad \text{and} \circlearrowright$$

$$[L^2, L_z] = 0 \quad \text{and } x \text{ and } y.$$

$$[L_z, L_\pm] = \pm\hbar(L_x \pm i L_y) = \pm\hbar L_\pm$$

- We proved: If

$$L^2 f = \lambda f \quad \text{and} \quad L_z f = \mu f$$

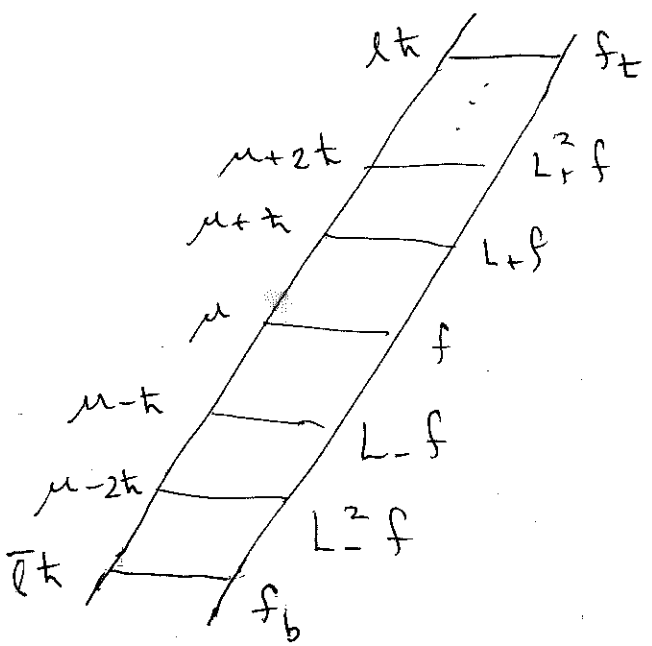
then

$$\boxed{L^2(L_\pm f) = \lambda(L_\pm f)}$$

and

$$\boxed{L_z(L_\pm f) = (\mu \pm \hbar)(L_\pm f)}$$

So L_+ raises the e-value from μ to $\mu+\hbar$ and L_- lowers it to $\mu-\hbar$, but they both leave L^2 alone.



$$= L_x^2 + L_y^2 \mp i(L_x L_y - L_y L_x)$$

$$= L_x^2 + L_y^2 \mp i(i\hbar L_z)$$

$$= L^2 - L_z^2 \pm \hbar L_z$$

Solving for L^2,

$$\boxed{L^2 = L_\pm L_\mp + L_z^2 \mp \hbar L_z}$$

It follows that

$$L^2 f_t = (L_- L_+ + L_z^2 + \hbar L_z) f_t$$

But we can't raise L_z indefinitely so eventually we hit a top rung

$$L_+ f_t = 0$$

Let $L_z f_t = l\hbar f_t$ and $L^2 f_t = \lambda f_t$

To relate l and λ lets try to find a relation btwn L^2 and $L_\pm$, L_z,

$$L_\pm L_\mp = (L_x \pm i L_y)(L_x \mp i L_y)$$

$$= (0 + \hbar^2 l^2 + \hbar^2 l) f_t = \hbar^2 l(l+1) f_t$$

So

$$\boxed{\lambda = \hbar^2 l(l+1)}$$

Similarly $L_- f_b = 0$ and

$$L^2 f_b = (L_+ L_- + L_z^2 - \hbar L_z) f_b$$

$$= (0 + \hbar^2 \bar{l}^2 - \hbar^2 \bar{l}) f_b = \hbar^2 \bar{l}(\bar{l}-1) f_b$$

Then $\lambda = \hbar^2 \bar{l}(\bar{l}-1) \Rightarrow \bar{l}(\bar{l}-1) = l(l+1)$

So,

$$\bar{l} = \begin{cases} l+1 & \text{absurd} \\ \underline{-l} & \text{yes!} \end{cases}$$

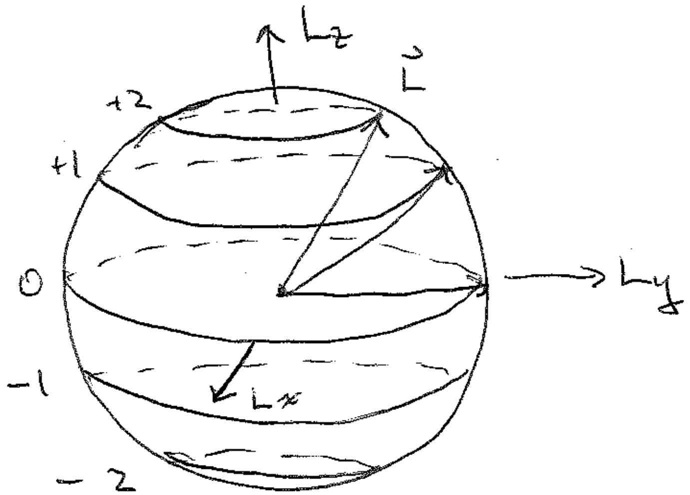
So the e-values of L_z are

$$m\hbar$$

where $m = -l, -l+1, \dots, l-1, l$ in integer steps. We must have

$$l = -l + N \quad \text{with } N \text{ an integer}$$

Rough Schematic



The radius $\sqrt{l(l+1)}$ is, in general greater than $\max(L_z)$!

So

$$l = \frac{N}{2}$$

is either an <u>integer or a half-integer</u>.

$$L^2 f_l^m = \hbar^2 l(l+1) f_l^m$$

and

$$L_z f_l^m = \hbar m f_l^m$$

where

$$l = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \dots ; \quad m = -l, -l+1, \dots, l-1, l$$

You can't get $\vec{L}$ to point completely along L_z.

Loren: "Why can't I just pick axes to lie along $\vec{L}$?"

To do so you would have to know <u>all</u> three components of $\vec{L}$.