Quantum Mechanics

Lecture 26

I Last time

II Michael's guest lecture on spin angular momentum.

III Some exercises in spin physics

$$P_\ell^m(x) \equiv (1-x^2)^{|m|/2} \left(\frac{d}{dx}\right)^{|m|} P_\ell(x)$$

and

$$P_\ell(x) \equiv \frac{1}{2^\ell \ell!} \left(\frac{d}{dx}\right)^\ell (x^2-1)^\ell$$

is the ℓ^{th} Legendre polynomial.

Then

$$Y_\ell^m(\theta,\phi) = \Phi\,\Theta = \epsilon \sqrt{\frac{(2\ell+1)(\ell-|m|)!}{4\pi\,(\ell+|m|)!}}\, e^{im\phi} P_\ell^m(\cos\theta).$$

and $\epsilon = (-1)^m$ for $m \geq 0$ and $\epsilon = 1$ for $m \leq 0$.

I. We found

$$\Phi(\phi) = e^{im\phi}$$

and

$$\Theta(\theta) = A\, P_\ell^m(\cos\theta)$$

the "associated Legendre function"

where

are the "Spherical harmonics" that play a central role in atomic physics.

These are automatically orthogonal (why?). So

$$\int_0^{2\pi}\int_0^{\pi} [Y_\ell^m(\theta,\phi)]^* [Y_{\ell'}^{m'}(\theta,\phi)] \times \sin\theta\, d\theta\, d\phi = \delta_{\ell\ell'}\delta_{mm'}$$

II See lecture notes.

III We have a new addition to our memory hall of fame — the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\chi = \frac{i}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

What is the probability of measuring $+\hbar/2$ ~~with the~~ for S_y in this state? What is $\langle S_+ \rangle$?

- If there's time G. 4.26 (b).

These matrices are traceless.

I like Lubsen's "the minus i rides high in σ_y".

- Derive another one of the spin matrices
- If you are given the spin state