Quantum Mechanics April 8th, 2015 P1/3

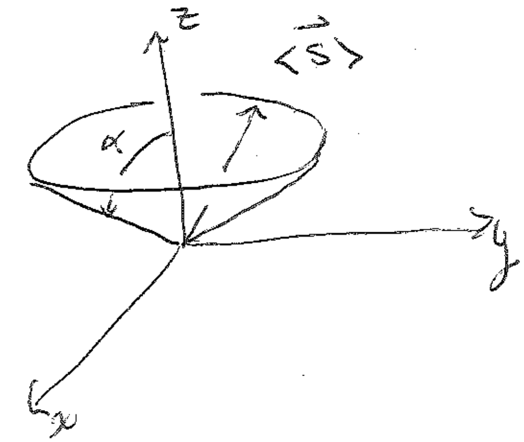
Lecture 29

I Last time

II Quantum Stern-Gerlach derivation

III SPINS simulation

I • Derived Larmor precession



$\langle S_x \rangle = \frac{\hbar}{2} \sin\alpha \cos(\gamma B_0 t)$

$\langle S_y \rangle = -\frac{\hbar}{2} \sin\alpha \sin(\gamma B_0 t)$

$\langle S_z \rangle = \frac{\hbar}{2} \cos\alpha$

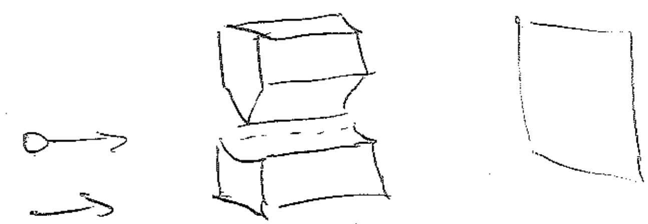
Larmor frequency $\omega = \gamma B_0$

• We saw that classically

$F_z = \gamma \alpha S_z$

• We looked at an animation and data showing the results of the Stern-Gerlach experiment

II



Move with silver atom

$$H(t) = \begin{cases} 0, & t < 0 \\ -\gamma(B_0 + \alpha z) S_z, & 0 \le t \le T \\ 0, & t > T \end{cases}$$

(Ignore B_x here, not significant)

Let $\chi(t) = a\chi_+ + b\chi_-$ for $t \le 0$

For $0 \le t \le T$

$$\chi(t) = a\chi_+ e^{-iE_+ t/\hbar} + b\chi_- e^{-iE_- t/\hbar}$$

with $E_\pm = \mp \gamma (B_0 + \alpha z)\frac{\hbar}{2}$.

So after time T

$$\chi(t) = \left(a e^{i\gamma T B_0/2}\chi_+\right) e^{i(\alpha\gamma T/2)z} + \left(b e^{-i\gamma T B_0/2}\chi_-\right) e^{-i(\alpha\gamma T/2)z}$$

for $T \ge T$

But recall that

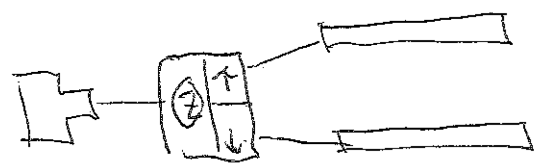
$$\psi_p(x) = e^{ipx/\hbar}$$

so the two terms each have momentum now! For the spin up term

$$p_z = +\frac{\alpha\gamma T\hbar}{2}$$

and so these silver atoms move upwards, while the 2nd term has negative momentum in the z-direction and so the beam splits!!

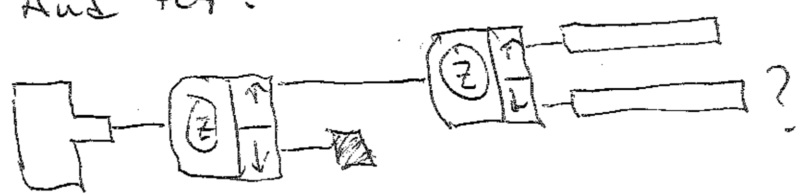
If we run the experiment



What should we get?

Predictions:

And for:



III The folks at Oregon State have put together a lovely simulation for testing your understanding of spin.

What about

and