Quantum Mechanics
Lect. III

Jan 30^{th}, 2015

I Last time
II Physical Example
III Interlude
IV Probability theory

I. • The heart of the difference between the 1-norm and the 2-norm is that ~~a~~ different sets of linear <u>transformations</u> preserve these two norms.

• In particular the transformations preserving the 2-norm can have negative entries and this allows for interference phenomena!

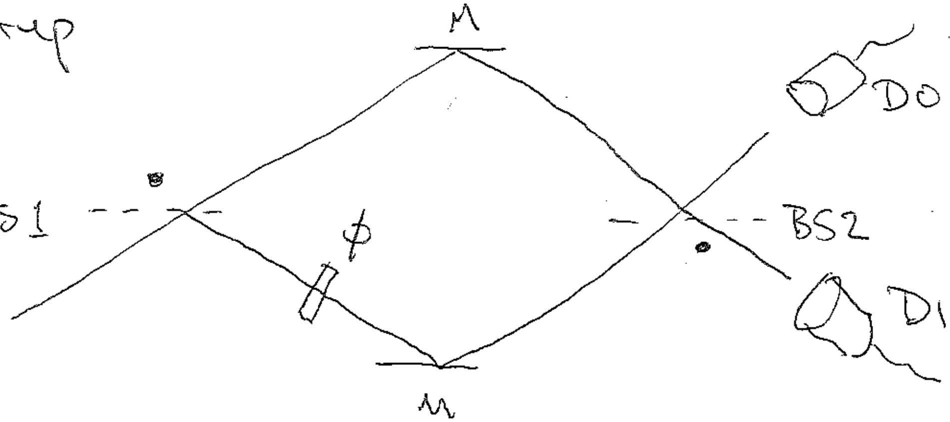
We didn't prove it but m-norms with $m > 2$ are even less interesting because they are only preserved by a finite set of transformations.

• We demonstrated inteference explicitly for a qubit

$$\begin{tikzcd}[column sep=small]
 & & & |0\rangle \arrow[dl] \arrow[dr] & & \text{flipping op} & \\
 & & |0\rangle \arrow[dl] \arrow[dr] & & |1\rangle \arrow[dl] \arrow[dr] & & \\
 & \cancel{|0\rangle} & & |1\rangle & & \cancel{|0\rangle} &
\end{tikzcd}$$

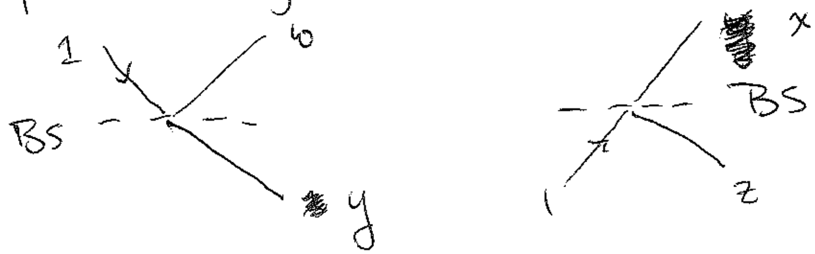
Let's revisit this with a more physical example

Let's return to the Mach-Zehnder setup



Here the beam splitter acts as a physical transformation of the input state. If BS1 is a half-silvered

of the beam splitter's action? ~~on a photon injected from below?~~



$$B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

But we know it causes 50-50 probabilities so $B\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}w\\y\end{pmatrix}$ and $|w|^2 = |y|^2 = 1/2$

mirror it lets through half the light and reflects the other half.

Suppose we represent putting a photon in from below by $\begin{pmatrix}0\\1\end{pmatrix}$ and from above by $\begin{pmatrix}1\\0\end{pmatrix}$.

What is the matrix representation

similarly

$$B\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}x\\z\end{pmatrix} \quad \text{and so}$$

$$|x|^2 = |z|^2 = 1/2$$

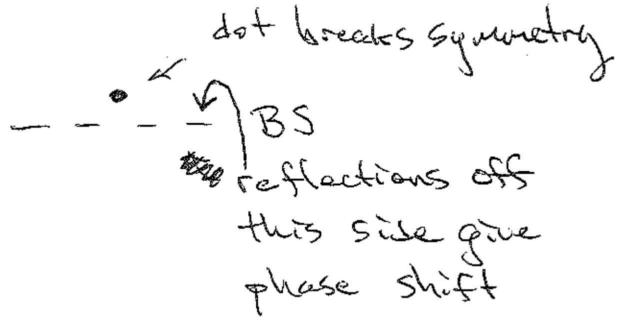
Finally we know we want the matrix to be orthogonal to really preserve the 2-norm so

$$B_\ell = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

because lower beam gets -1 phase.

Physically, the $-i$ phase shift is because the reflection off a mirror causes such a phase shift.

To make this asymmetry explicit we draw



So <u>only</u> the upper detector sees a click! Experiments confirm this prediction of quantum mechanics!!

III Interlude: Could Q.M. be different?

There are two more mysterious seeming aspects of Q.M.

Not only do we use the 2-norm,

The matrix for

is

$$B_u = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

So, if we send in $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ which detector(s) see a click(s).

$$B_\ell B_u \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

but we allow the amplitudes $\alpha, \beta, \ldots$ to be complex numbers! $\alpha, \beta, \ldots \in \mathbb{C}$. This is why the absolute values are essential

$$|\alpha|^2 + |\beta|^2 = 1.$$

Mystery # 1: Why are quantum amplitudes complex?

The second mystery is an unstated assumption in all of our discussion so far

Mystery #2: Why have we restricted attention to only <u>linear</u> transformations that preserve the 2-norm?

My current favorite answers to these questions are better suited to discussions later in the course; but I hope you will ponder them yourselves as, particularly my answer to ~~M~~ Mystery #1, mostly just trades it for another mystery.

<u>IV</u> The foundation of probability theory is just counting — no need for intimidation!

The surprising thing is that counting can be hard.

Discrete Systems with a finite total number of possibilities are simplest, then the probability is just the ratio of two counts:

$$P(\text{event } e) = \frac{\#\text{ of events } e}{\text{total }\#\text{ of events}}$$

↖ "number"

<u>Ex</u>: A bag contains 3 red marbles, 2 black and 1 white. What's the probability of drawing a red?

$$P(\text{red}) = \frac{3 \text{ reds}}{6 \text{ total}} = \frac{1}{2}$$

Easy (but important) coherence check: if you draw a marble what's the probability of drawing a marble?

$$P(\text{marble}) = 1 = P(\text{red}) + P(\text{black}) + P(\text{white})$$

boring

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

more interesting.

Exposes relationships btwn events (types)

Mr. Jones has two children. The older child is a girl. What is the probability that both children are girls?

Mr Smith has two children. At least one of them is a boy. What is the probability that both children are boys?