Quantum Mechanics April 15th, 2015 P1/2

Lect. 32

I. Last time

II. Maya guest lecture

III Power series Solution of the harmonic oscillator

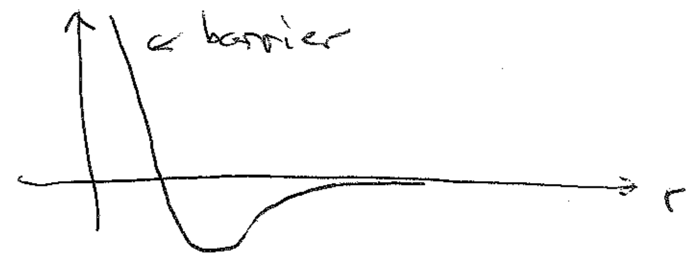
• H atom Structure and history — Maya will discuss at more length momentarily

• Began to solve the radial equation by series.

Recall $u(r) = r\,R(r)$

I • Effective potential and the centrifugal barrier

$$V_{eff} = V(r) + \frac{\hbar^2 l(l+1)}{2mr^2}$$



led to

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[-\frac{e^2}{4\pi\epsilon_0}\frac{1}{r} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = Eu$$

Then $k = \frac{\sqrt{-2mE}}{\hbar}$, $\rho = kr$, $\rho_0 = \frac{me^2}{2\pi\epsilon_0\hbar^2 k}$

gives $\frac{d^2u}{d\rho^2} = \left[1 - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2}\right]u$

Attempt $u(\rho) = \rho^{l+1} e^{-\rho} v(\rho)$, with $v(\rho) = \sum_{j=0}^{\infty} c_j \rho^j$

II See notes for Maya's guest lecture

III For the harmonic oscillator

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\psi = E\psi$$

a useful shorthand is

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$$

and we get

$$\frac{d^2\psi}{d\xi^2} = (\xi^2 - K)\psi$$

with

$$K \equiv \frac{2E}{\hbar\omega},$$

Your job is to solve this by series and find the allowed energies.

For large ξ we get,

$$\frac{d^2\psi}{d\xi^2} \approx \xi^2\psi$$

and has solutions

$$\psi(\xi) \approx A e^{-\xi^2/2} + B \overset{0}{\cancel{e^{+\xi^2/2}}}$$

So we want solutions of the form

$$\psi(\xi) = h(\xi)\, e^{-\xi^2/2}$$

Then,

$$\frac{d\psi}{d\xi} = \left(\frac{dh}{d\xi} - \xi h\right) e^{-\xi^2/2}$$

and

$$\frac{d^2\psi}{d\xi^2} = \left(\frac{d^2h}{d\xi^2} - 2\xi\frac{dh}{d\xi} + (\xi^2 - 1)h\right) \times e^{-\xi^2/2}$$

Then, Sch. eqn is

$$\frac{d^2h}{d\xi^2} - 2\xi\frac{dh}{d\xi} + (K-1)h = 0$$