Quantum Mechanics

Lect. 4

I Last time

II A bit more probability

III What's to come?

IV Fermat's principle

I
- Explored a physical example with quantum interference — the Mach-Zehnder interferometer
- The 2-norm is what allows amplitudes to take center stage and, hence, makes interference possible.

Left open the mysteries of complex #'s and linearity.

- Began probability theory

$$P(\text{event } e) = \frac{\#\text{ of events } e}{\text{total }\#\text{ of events}}$$

$$\sum_i P_i = 1$$

II In a class of 3 students a particularly difficult exam is given with the results

student	score (100)
S1	42
S2	57
S3	~~[illegible]~~ 42

call it j

What is the mean score?

notation for average

$$\langle j \rangle = \frac{42+57+42}{3} = \frac{141}{3} = 47.$$

With measurements in mind, in quantum we call this the "expectation value". You have clear intuition for this, but let's rewrite it in a general form

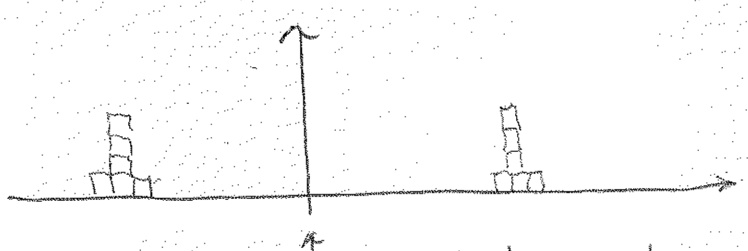
$$\langle j \rangle = \cancel{5\!\!\!\!\!/}\,57 \cdot \frac{1}{3} + 42 \cdot \frac{2}{3}$$

← prob. of score 57

← prob. of score 42

$$= \sum_j j\, P(j)$$

← probability of score j



We would never measure the expected value; this is why it is often nice to characterize the spread of your data

Let's try $\langle \Delta j \rangle = \langle j - \langle j \rangle \rangle$

This last formula works for any kind of variable

$$\langle \heartsuit \rangle = \sum_{\heartsuit\text{'s}} \heartsuit\, P(\heartsuit)$$

So,

$$\langle j^2 \rangle = \sum_{j\text{'s}} j^2 P(j)$$

Notice the ~~difficulties~~ difficulty of expectation values in the following graph:

where $\Delta j \equiv j - \Delta j$. This is

$$\langle \Delta j \rangle = \sum_j (j - \langle j \rangle) P(j)$$

$$= \sum_j j P(j) - \sum_j \langle j \rangle P(j)$$

$$= \langle j \rangle - \langle j \rangle \sum_j P(j) = 0 \, !$$

Won't work. Try again: $\langle (\Delta j)^2 \rangle$ is awesome! "Standard deviation σ"

$$\sigma^2 \equiv \langle (\Delta j)^2 \rangle = \sum_j (j - \langle j \rangle)^2 P(j)$$

$$= \sum_j (j^2 - 2j\langle j\rangle + \langle j\rangle^2) P(j)$$

$$= \sum_j j^2 P(j) - 2\langle j\rangle \sum_j j\, P(j) + \langle j\rangle^2 \sum_j P(j)$$

$$= \langle j^2\rangle - 2\langle j\rangle\langle j\rangle + \langle j\rangle^2$$

$$= \langle j^2\rangle - \langle j\rangle^2$$

by far the easiest way to compute σ.

That's it, for (most) practical calculations we're only going to need $\langle j\rangle$, σ, and $P(j)$.

However, we do need one more technique; that is, the ability to deal with continuous probability distributions.

Suppose you want to know how probable your height was, how would you go about this?

You would grab a tape measure and find your height, but only with limited precision. Let's say it is $5'8'' \pm \frac{1}{16}''$ then how probable is this?

To answer that question we need a function that tells us the likelihood of different heights and we need to integrate it:

$$P(\text{your height}) = \int_{5'8''-\frac{1}{16}''}^{5'8''+\frac{1}{16}''} \rho(h)\, dh$$

carries units of height!!

But, if dh has units $[dh] =$ length (here feet and inches) what does that mean about $\rho(h)$? It also must carry units

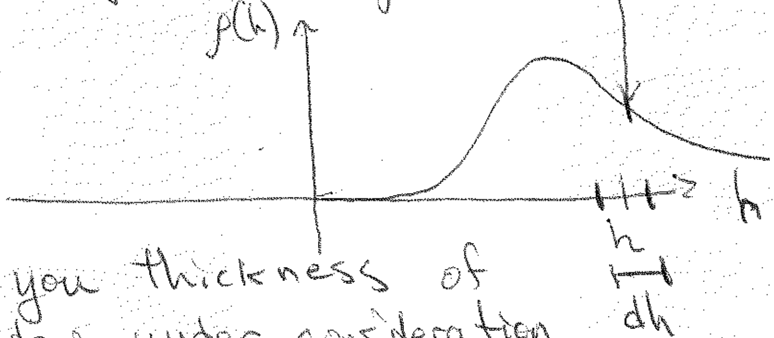
$$[\rho(h)] = \frac{\text{Probability}}{\text{length}}$$

This denominator is why we call it a Probability density.

So,

$$\rho(h)\,dh = \left\{ \begin{matrix} \text{Probability that an individual} \\ \text{(chosen at random) lies btwn } h \text{ and } h+dh \end{matrix} \right\}$$

tells you where you are



tells you thickness of window under consideration.

Once you understand this, everything else is quite similar:

the wave function $\Psi(x)$ is the analog of the amplitudes α, β etc, that we have been considering; that is,

$$\rho(x) = |\Psi(x)|^2$$

$$= \Psi^*(x) \cdot \Psi(x).$$

III What's to come?

Survey class: Who has taken

$$\int_{-\infty}^{\infty} \rho(x)\,dx = 1$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,\rho(x)\,dx$$

or $f(x)$

$$\langle \heartsuit(x) \rangle = \int_{-\infty}^{\infty} \heartsuit(x)\,\rho(x)\,dx$$

$$\sigma^2 \equiv \langle (\Delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

As you may remember

classical mechanics? 2

Who knows what a Hamiltonian is? 3

Who has heard of Hamilton's principle or Fermat's principle? 5

That light takes the path of least time connecting two points? ___

My plan is to give you a brief introduction to where quantum mechanics came from in the next 3 lectures.

This is not for culture or because it's "good for you" to know the history.

Instead I want to try and motivate that many unintuitive ideas in quantum didn't come out of nowhere, but were motivated by classical mechanics. I want you to have some physical insight into the definitions we will constantly use.

IV Fermat's principle

Light moving in a vacuum travels at a constant speen c. In a medium with index of refraction n the speed is

$$v = \frac{c}{n}.$$

In a vacuum if we send light from a source pt. S to a detector at pt. D it travels in a straight line:

What if the detector is embedded in a medium, say glass, then what is the path of the ray?

The answer is familiar from Snell's law, but Fermat gave a very interesting formulation.

<u>Fermat's</u> <u>principle</u>:

Light travels the path from S to D that is extremal in time.