# Quantum Mechanics
## Lect. 5



I Last time

II Fermat's principle

III Huygens' principle and wave theory

IV The opto-mechanical analogy

I • $\langle f(\heartsuit)\rangle = \sum_{\heartsuit\text{'s}} f(\heartsuit)\,P(\heartsuit)$

$$\sigma^2 = \langle(\Delta j)^2\rangle = \langle j^2\rangle - \langle j\rangle^2$$

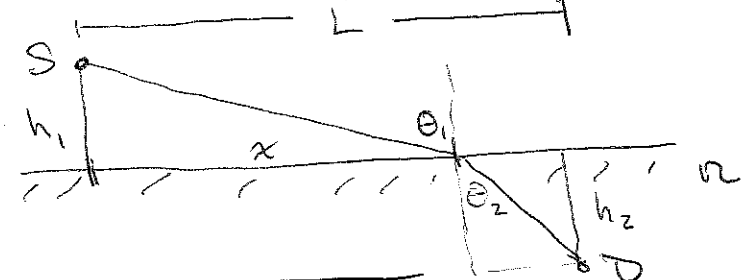
• Probability density $\rho(x)$

$$[\rho(x)] = \frac{\text{Probability}}{\text{Units of } x}$$ make it a density

$$\rho(x)\,dx = \left\{ \begin{array}{l} \text{Prob. of event} \\ \text{btwn } x \text{ and } x+dx \end{array} \right\}$$

• <u>Fermat's principle</u>: Light travels the path from S to D that is extremal in time.

II Last time we started to investigate an example of Fermat's principle; we considered a ray going from a Source S in air to a detector D in glass of index of refraction $n$.




$$t_1 = \frac{\sqrt{h_1^2 + x^2}}{c}$$

$$t_2 = \frac{\sqrt{h_2^2 + (L-x)^2}}{c/n}$$

so,

$$T = t_1 + t_2 = \frac{\sqrt{h_1^2 + x^2}}{c} + \frac{n\sqrt{h_2^2 + (L-x)^2}}{c}$$

We are looking for an extremum,

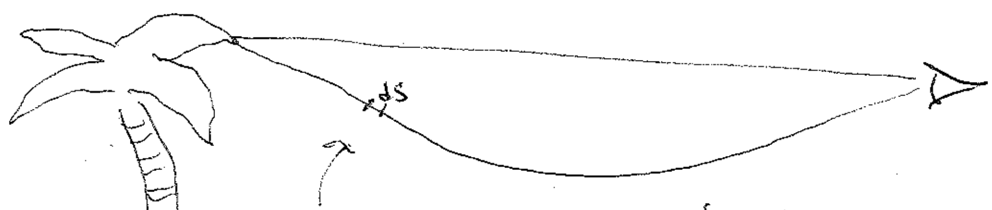
so,

$$\frac{dT}{dx} = 0 = \frac{1}{2}\frac{2x}{c\sqrt{h_1^2 + x^2}} - \frac{1}{2}\frac{n2(L-x)}{c\sqrt{h_2^2+(L-x)^2}}$$

$$\Rightarrow \boxed{\sin\theta_1 = n\sin\theta_2} \quad \text{Snell's law.}$$

Of* course, the index of refraction interface need not be so simple, e.g. $n$ could vary continuously $n = n(h)$.

different paths




This ray looks as though it's been reflected off of water.

Let's set up the general case

$$T = \int_{t_0}^{t_1} dt = \frac{c}{c}\int_{t_0}^{t_1} \frac{dt}{ds}\cdot\frac{ds}{dt}\,dt$$

small arc length

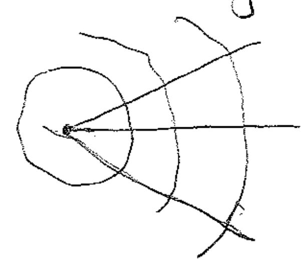
In fact, this explains many desert mirages.— the index of refraction of air is not exactly one and varies with temperature. This allows light leaving a palm tree to reach your eye along two

$$= \frac{1}{c}\int_S^D \frac{c}{v}\,ds = \boxed{\frac{1}{c}\int_S^D n\,ds}$$

So, in general you try to find the path connecting S and D that ~~minimizes~~ extremizes this integral

$$\boxed{T = \frac{1}{c}\int_S^D n\,ds}.$$

But, we've all been taught that the wave theory of light won out, and so, these rays are fictitious. Really there are wave fronts and rays are just a nice guideline overlay



So, is Fermat's principle a mathematical curiosity? A "divine miracle"? No! P3/4

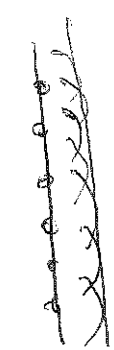
Actually it follows most clearly from the wave theory.

III Huygens' principle

Our goal here is not to develop in full the wave theory of light, but let's pull in one of its beautiful principles.

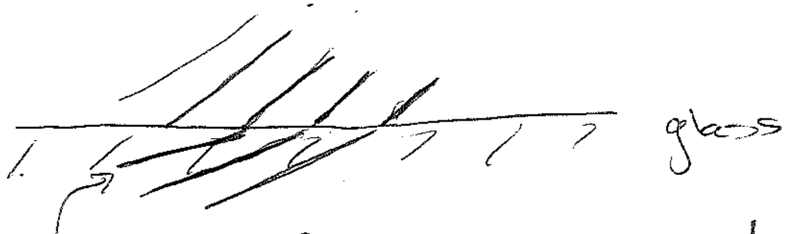
If you're given a wave front, say of a plane wave, how do you construct the next one in the train?

Huygen's gave an answer: break the front into a (large) collection of point sources, the next front is given by the superposition of spherical waves emitting from these sources



Notice the clean role of locality in this construction.

The Fermat principle follows simply ~~from~~ from Huygens' principle. Let's take an example again.




Because the local speed changes at the glass interface the wave front bends. This due to wavelets in the air rapidly catching up to those in the glass.

A useful analogy is a marching band ~~a~~ marching across a partially dry ~~field~~ and partially muddy field. Marchers entering the mud slow and the other marchers cover more ground, causing the wave front to bend.

IV Next time we will discuss Hamilton's remarkable observation that there is a formulation of mechanics quite similar to Fermat's formulation of optics.