Quantum Mechanics Feb $13^{th}$, 2015 P1/4

Lect 9

I Last time

II States with definite measurement outcomes (Determinate States)

transformations of the states

→ this requires linear algebra!

• Notation

$$|\alpha\rangle \rightarrow \vec{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix}$$

abstract (useful for conceptual calculations)

concrete (useful for getting concrete answers)

I • Adrian asked the question "why linear algebra?"

Quantum theory

Operators (matrix operators)

wave functions (also vectors of amplitudes)

The operators act as linear

$$\langle \alpha | \beta \rangle = a_1^* b_1 + \cdots + a_N^* b_N$$

$$|\beta\rangle = T|\alpha\rangle \rightarrow \vec{b} = \overleftrightarrow{T}\vec{a}$$
$$= T\vec{a}$$
$$= \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Also $|\Psi\rangle \rightarrow \Psi(x)$

$$\langle f | g \rangle = \int f^* g \, dx$$

$$|g\rangle = \hat{Q}|f\rangle \rightarrow g(x) = \hat{Q}f(x).$$

- Introduced Hilbert Space

{Vector space of all functions of $x$} $= \{f(x)\} = V$

But we only want normalizable states; $\Psi$ s.t.

$$\int |\Psi|^2 dx = 1$$

So introduce

Hilbert space

$$\mathcal{H} = \mathbb{L}_2(a,b)$$

$= \{$ space of all functions $f$ s.t.

$$\int_a^b |f(x)|^2 dx < \infty \}.$$

and

$$\langle f_m | f_n \rangle = \delta_{mn} \quad \text{orthonormal}$$

A set of functions $\{f_n(x)\}$ is $\underline{\text{complete}}$ if any function $f(x)$ can be written

$$f(x) = \sum_{n=1}^{\infty} c_n f_n(x).$$

and Fourier's trick:

$$c_n = \langle f_n | f \rangle.$$

- Introduced terminology & Fourier's trick

$$\langle f_m | f_n \rangle = 0 \quad \text{orthogonal}$$

$$\langle f_n | f_n \rangle = 1 \quad \text{normalized}$$

works if $\{f_n\}$ is orthonormal

- Defined hermiticity

$$\langle f | \hat{Q} f \rangle = \langle \hat{Q} f | f \rangle \quad \forall f(x)$$

for all

Equivalent to (HW)

$$\langle f | \hat{Q} g \rangle = \langle \hat{Q} f | g \rangle \quad \forall f, g.$$

Physically significant because every measurement outcome is real.

This implies that $\langle Q \rangle$ is also,

$$\langle Q \rangle = \langle Q \rangle^*.$$

$$\boxed{\text{Observables are represented by Hermitian operators}}$$

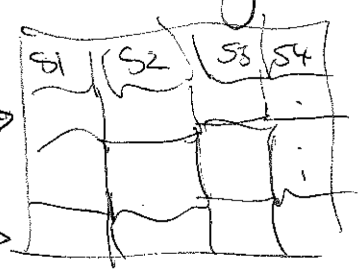
One more characterization of hermiticity uses the <u>adjoint</u>. By definition $\hat{Q}^\dagger$ is the operator s.t.

$$\langle f | \hat{Q} g \rangle = \langle \hat{Q}^\dagger f | g \rangle \quad \forall f, g.$$

So another way to characterize hermiticity is

$$\boxed{\hat{Q}^\dagger = \hat{Q}.}$$

Think of Hollywood squares




Measure this one $\rightarrow$ get one outcome

Measure this one $\rightarrow$ get a different outcome in general!

Describe measurements statistically!

II Ensemble = { Huge # of identically prepared states }

But, are there some states that have definite outcomes, say when you measure Q? Determinate states of Q?

We'd certainly $\sigma^2 = 0$

$$\sigma^2 = \langle (\hat{Q} - \langle \hat{Q} \rangle)^2 \rangle$$

$$= \langle \Psi | (\hat{Q} - \langle \hat{Q} \rangle)^2 \Psi \rangle$$

$$= \langle \Psi | (\hat{Q}-q)^2 \Psi \rangle$$

$$= \langle (\hat{Q}-q)\Psi | (\hat{Q}-q)\Psi \rangle = 0$$

But, the only state that is ~~orth~~ $\perp$ to ~~it~~self is ~~Bas~~ zero, so

$$(\hat{Q}-q)\Psi = 0 \Rightarrow \boxed{\hat{Q}\Psi = q\Psi}$$

This is the eigenvalue eqn.

$q$ is said to be the eigenvalue or e-value.

$\Psi$ is the eigenfunction or the e-function.

$$\boxed{\begin{array}{c}\text{Determinate States} \\ \text{are} \\ \text{eigenfunctions of } \hat{Q}\end{array}}$$