Homework 1

Due by 10pm on Wednesday, February 5th, 2020

Reading: Griffiths Chap. 1, sections 1.1-3. Class notes.

1. In class we defined orthogonal matrices in two ways. One definition was as the set of all matrices O that preserved the 2-norm. The second definition was as the set of matrices O that satisfied

$$\widetilde{\mathsf{O}} = \mathsf{O}^{-1}, \tag{1}$$

where the tilde stands for the matrix transpose. Show that these two definitions are equivalent. Feel free to restrict your attention to the 2×2 case if you prefer. There are many different ways of doing this problem. If you're struggling to find a short proof, don't hesitate to grind the proof out by writing out the above definitions in terms of the components of the 2×2 matrices. This is still an involved argument.

2. Griffiths Problem 1.1.
3. Griffiths Problem 1.2.

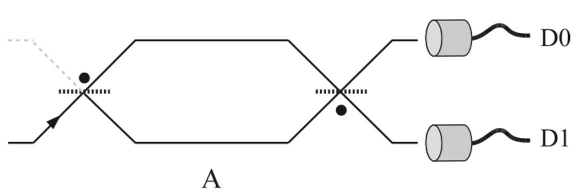
4. Memorize the integral of a Gaussian:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}. \tag{2}$$

You can simply mark this problem as done when you turn in your homework. If you don't know any tools for memorizing things, just ask me. There are plenty of things that make it easier. I will follow up with a quiz on this at some point in the future.

5. Griffiths Problem 1.3.

6. Consider the simplified Mach-Zehnder interferrometer shown below. Introduce a photon along



the lower beam, so that the input amplitude vector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Make a table of the possible outcomes and their probabilities for this setup.

7. Using the setup of the last problem, now suppose that you put a photographic plate at the point A in the lower beam. Once again make a table of possible outcomes (now there are three) and their probabilities for this setup.

8. Avshalom Elitzur and Lev Vaidman (1993) used the setup of the last two problems in a creative and surprising way. They imagined a factory that produces a type of firework triggered by light.[1] So sensitive is the trigger that the passage of a single photon through its mechanism will explode a firework.

Because of manufacturing defects, however, many fireworks come off the assembly line without working triggers. Photons pass through these mechanisms without being registered at all, and the fireworks are duds. The factory managers want to be able tell for sure that at least some fireworks are in working order. How can they do this? Of course, if they send a photon through a given firework, and it blows up, then they can be sure that the firework was in working order—but they have also destroyed that firework. What the managers want is a way to identify fireworks that are explosive, but are not yet exploded. Since the firework triggers are set off even by one photon, this appears impossible.

a) Explain how the interferometer of the last two problems can be used to do the job.
b) Suppose the interferometer test is performed on a large number of fireworks from the factory. When the test is inconclusive on a particular firework, it is repeated until the fireworks status is reasonably certain. What fraction of the working fireworks are certified as working but not detonated?

[1]They actually used bombs in their thought experiment. I prefer the less violent, but equally dramatic firework.