Homework 3

Due by 10pm on Wednesday, February 19th, 2020

Reading: Griffiths & Schroeter Chap. 3, sections 3.1-3. Appendix A.1-A.2. Class notes.

1. G&S Problem 1.11.
2. In class we have discussed Fourier series as an example of viewing functions as vectors and for illustrating bases of functions. However, I skipped over plenty of details that justify some claims. Let's go through some of these here. First let's explore orthonormality. (a) By hand show that:

$$\left\langle \sin\left(\frac{2\pi m}{L}x\right) \Big| \sin\left(\frac{2\pi n}{L}x\right) \right\rangle = \delta_{mn},$$

$$\left\langle \cos\left(\frac{2\pi m}{L}x\right) \Big| 1 \right\rangle = \delta_{m0} L,$$

and

$$\left\langle \cos\left(\frac{2\pi m}{L}x\right) \Big| \sin\left(\frac{2\pi n}{L}x\right) \right\rangle = 0, \qquad \text{for all } m \text{ and } n.$$

[Hint: You will probably want to use a trigonometric identity to simplify the integrands first and reduce these to sums of integrals you know how to do, as we did in class.]
(b) Now, assume that we have a function $f(x)$ that is periodic with period 2π and that it can be expanded as

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx). \tag{1}$$

(The fact that any such periodic function can be expanded this way is called **completeness** and we won't prove it here; it's a nice thing to tackle in a math class.) If I am given the function $f(x)$ how do I find a_0? Similarly prove one of the following formulas by hand:

$$\begin{aligned} a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx. \end{aligned} \tag{2}$$

(c) Find the Fourier series, that is, the a_0, a_n, and b_n, for the periodic function

$$f(x) = \frac{x}{\pi}, \qquad -\pi < x < \pi. \tag{3}$$

3. **Python Problem:** Using Python, plot $f(x) = x/\pi$, as well as your results for $a_0 + a_1\cos(x) + b_1\sin(x)$, $a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x)$ and so on including terms up to $n = 4$. The notebook on plotting that I have put on our computing tab should be helpful in getting used to plotting. Of course, ask for help troubleshooting as you need it.

4. G&S Problem 3.3.
5. G&S Problem 3.5 [skip part (c)].
6. G&S Appendix A, Problem A.2.
7. G&S Appendix A, Problem A.4.
8. Use the Gram-Schmidt procedure (see Problem A.4) to orthonormalize the functions $1, x, x^2$, and x^3 on the interval $-1 \le x \le 1$—they are (apart from the normalization) the **Legendre polynomials**, see Table 4.1 of Griffiths & Schroeter.