Homework 4

Due by 10pm on Wednesday, February 26th, 2020

Reading: Griffiths & Schroeter Chap. 2, sections 2.1-2.3.1 (skip 2.3.2 for now) Class notes.

Read the first half page of G&S section 2.5.2 to remind yourself of the definition of Dirac's delta function. [You don't need to go past Eq. (2.117).]
1. G&S Problem 2.22. [Hint: Be careful with part (c).]

In class and on the homework we have been discussing Fourier series. Last week, Henry raised the question of how Fourier series are related to the Fourier transform. There is a classic theorem in Fourier analysis, **Plancherel's Theorem**, that will be an immensely useful tool throughout the course and is closely related to Henry's question. The theorem states that

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{ikx} dk \iff F(k) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} f(x) e^{-ikx} dx.} \tag{1}$$

2. Griffiths Problem 2.19 guides you through a proof of this result. Unfortunately, this problem has several unexpected notation changes with respect to the conventions we have been using. I'm sorry! G&S cite Boas and so I thought they would follow her notation, but they do not! Denoting the notation that I have been using as $a_{n\text{Hal}}$ and G&S notation by $a_{n\text{GS}}$, etc., we have

$$\begin{aligned} a_{n\text{GS}} &= b_{n\text{Hal}}, \qquad \text{for } n = 1, 2, 3, \ldots, \\ b_{n\text{GS}} &= a_{n\text{Hal}}, \qquad \text{for } n = 1, 2, 3, \ldots, \\ b_{0\text{GS}} &= \frac{1}{2} a_{0\text{Hal}}, \quad \text{and} \quad L_{\text{Hal}} = 2a_{\text{GS}}. \end{aligned}$$

You should use the G&S notation for this problem, which is the notation established in the problem itself, and I hope you will accept my apologies for this annoying (and potentially confusing) notation change.
3. Griffiths Problem 2.26.

4. Griffiths Problem 2.4.
5. Griffiths Problem 2.5.
6. Griffiths Problem 2.7.
7. Griffiths Problem 1.12.