Today

I. Last Time

II. More Examples of the Energy-Time Uncertainty Relation

III. Bound and Scattering States in the Delta Function Potential

Last Time

*Energy-Time uncertainty: $\Delta E \Delta t \geq \frac{\hbar}{2}$

$$\Delta t = \frac{\sigma_Q}{\left| \frac{d\langle \hat{Q} \rangle}{dt} \right|}$$

$$\Delta E = \sigma_H$$

*Derived the uncertainty principle:

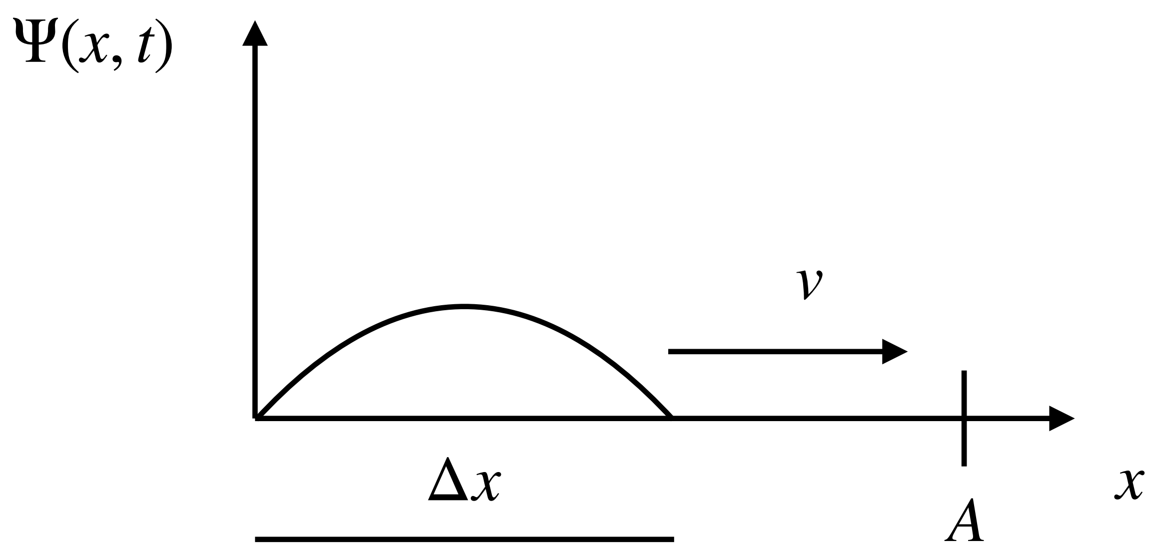
$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$

II. Examples of Energy-Time Uncertainty

Delta particle has a finite lifetime and hence has a spread in mass.

How long does it take a traveling wave packet to pass a pt. A?

Rough estimate:



$$\Delta t = \frac{\Delta x}{v} = \frac{m\Delta x}{p}$$

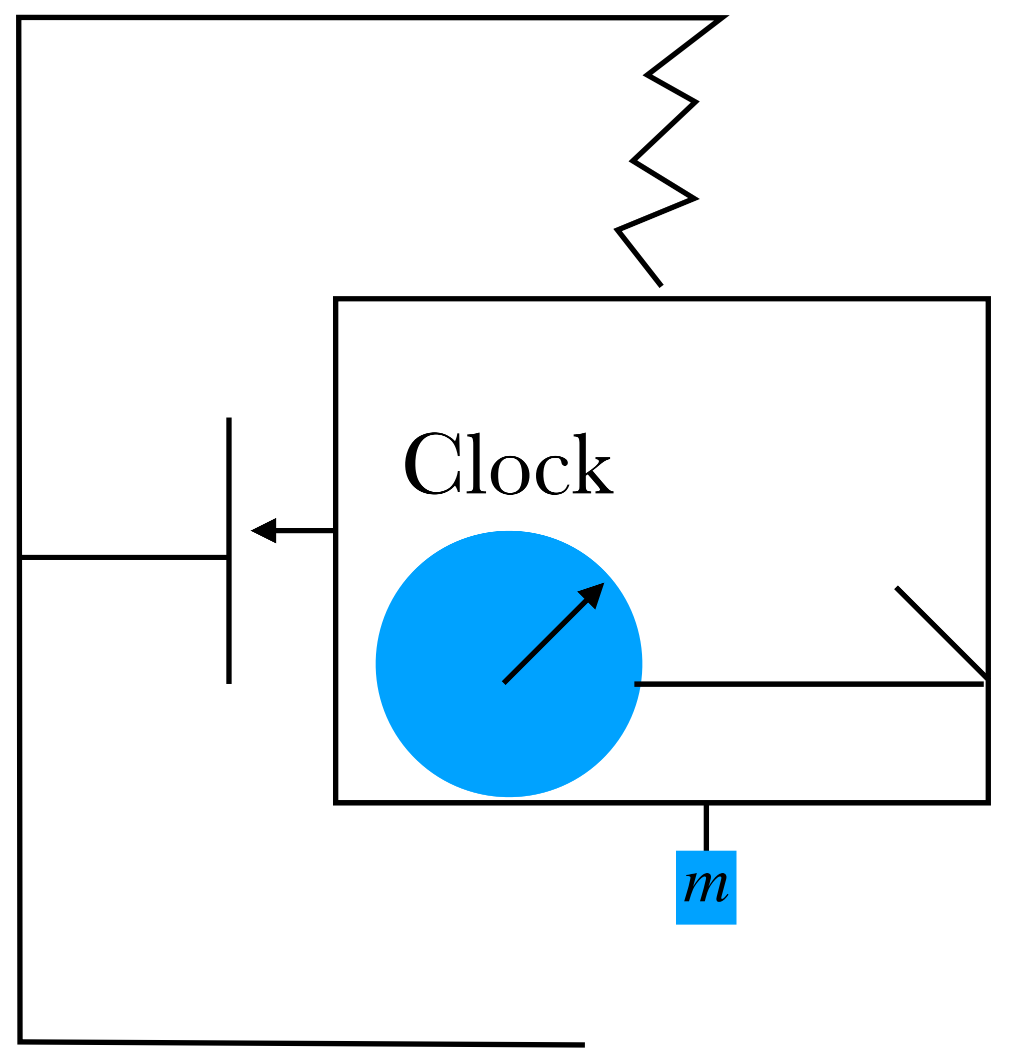
$$E = \frac{p^2}{2m} \qquad \Delta E = \frac{p\Delta p}{m}$$

$$\Delta E \Delta t = \frac{p\Delta p}{m}\frac{m\Delta x}{p} = \Delta p \Delta x \geq \frac{\hbar}{2}$$

There's a nice consistency between these two uncertainty principles.

Einstein proposed an experiment to refute such a relationship:

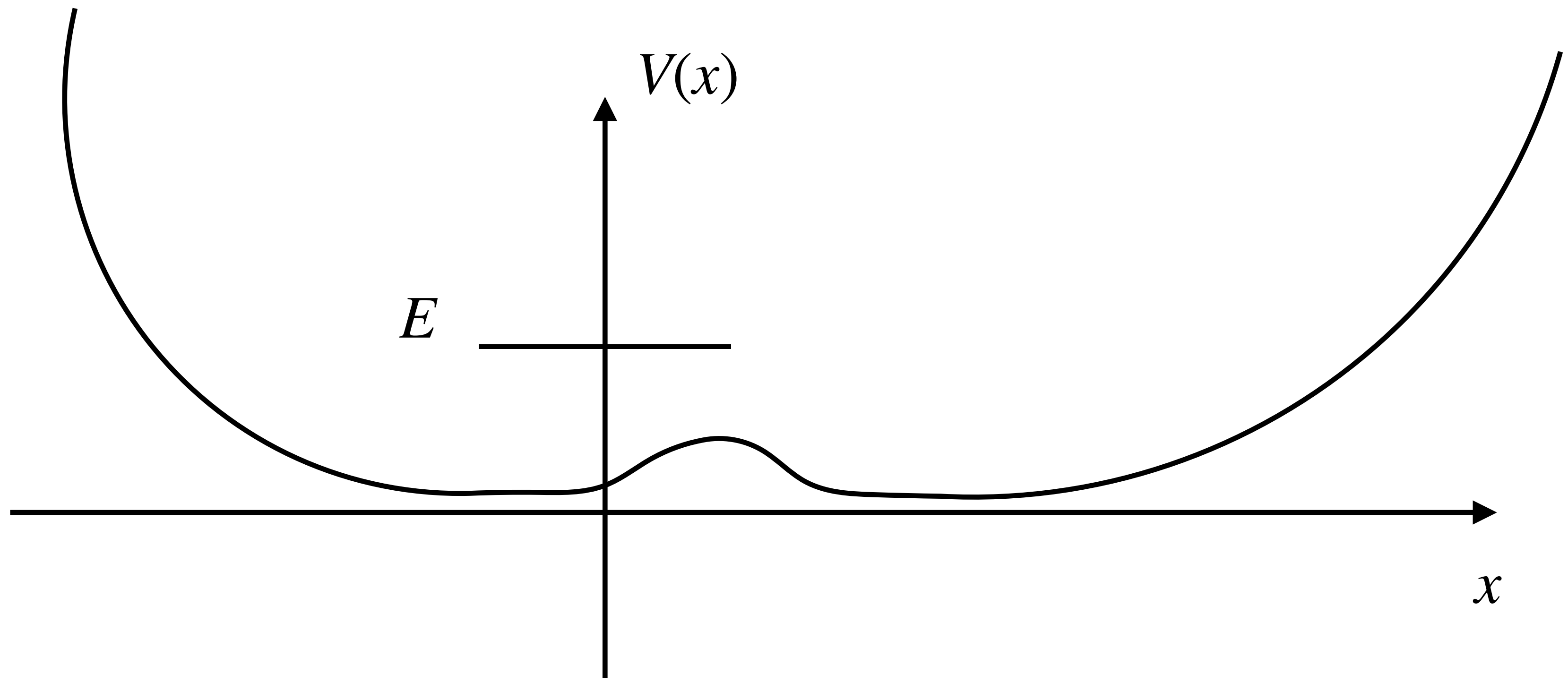
$$\Delta E \Delta t \geq \frac{\hbar}{2}$$



Bohr's argument again established the uncertainty principle.

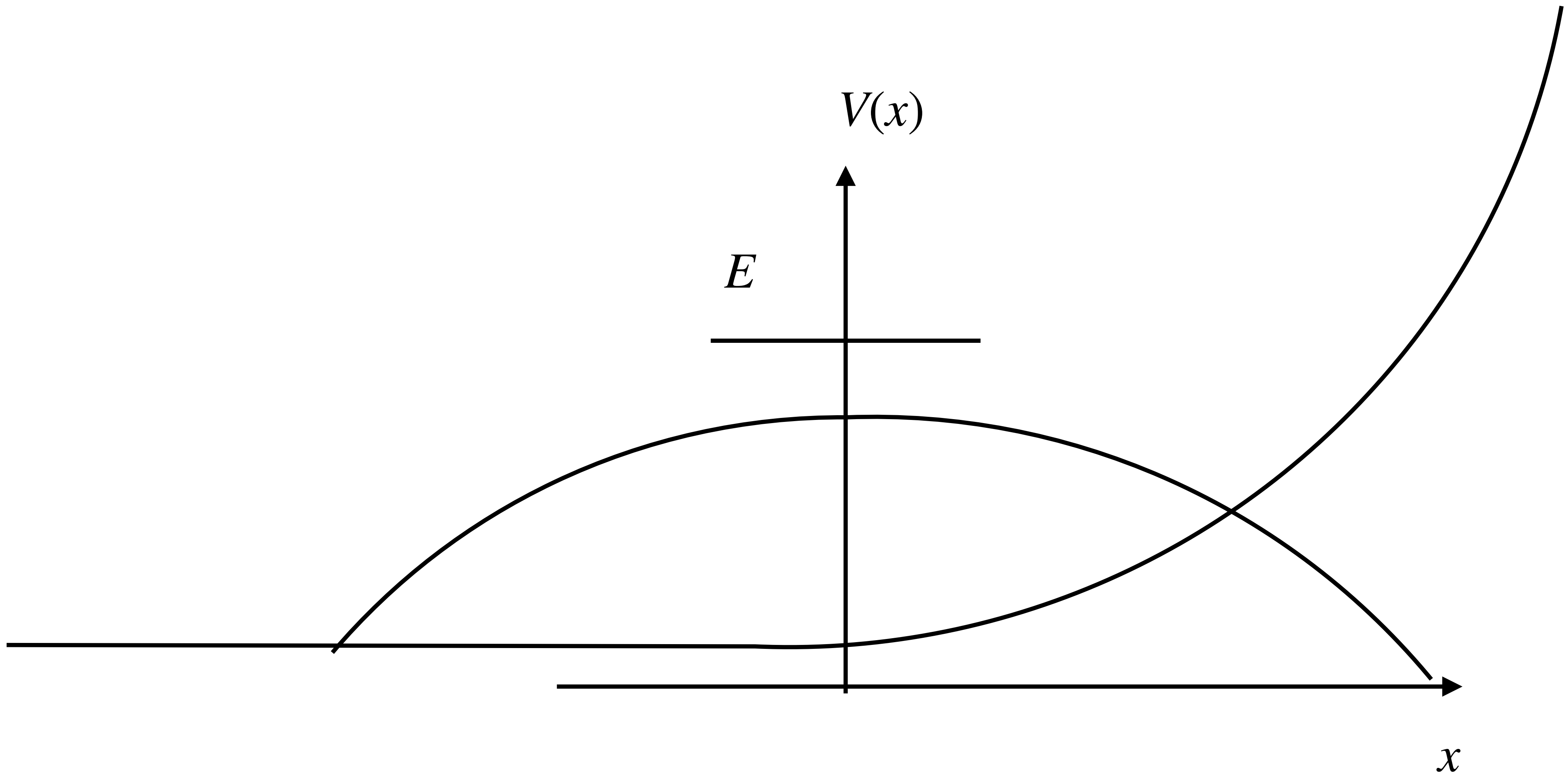
The wonderful quote from Rosenfeld: It was a real shock for Bohr...who, at first, could not think of a solution. For the entire evening he was extremely agitated, and he continued passing from one scientist to another, seeking to persuade them that it could not be the case, that it would have been the end of physics if Einstein were right; but he couldn't come up with any way to resolve the paradox. I will never forget the image of the two antagonists as they left the club: Einstein, with his tall and commanding figure, who walked tranquilly, with a mildly ironic smile, and Bohr who trotted along beside him, full of excitement...The morning after saw the triumph of Bohr.

III. Bound and Scattering States in the Delta Function Potential



Bound state: $E < V(-\infty)$ **and** $V(\infty)$

Scattering state: $E > V(-\infty)$ or/and $V(\infty)$



Examples: For the infinite square well can only support bound states
For the harmonic oscillator, again only bound states possible.

For the free particle, $V(x) = 0$, only scattering states.

Today: δ-function well.

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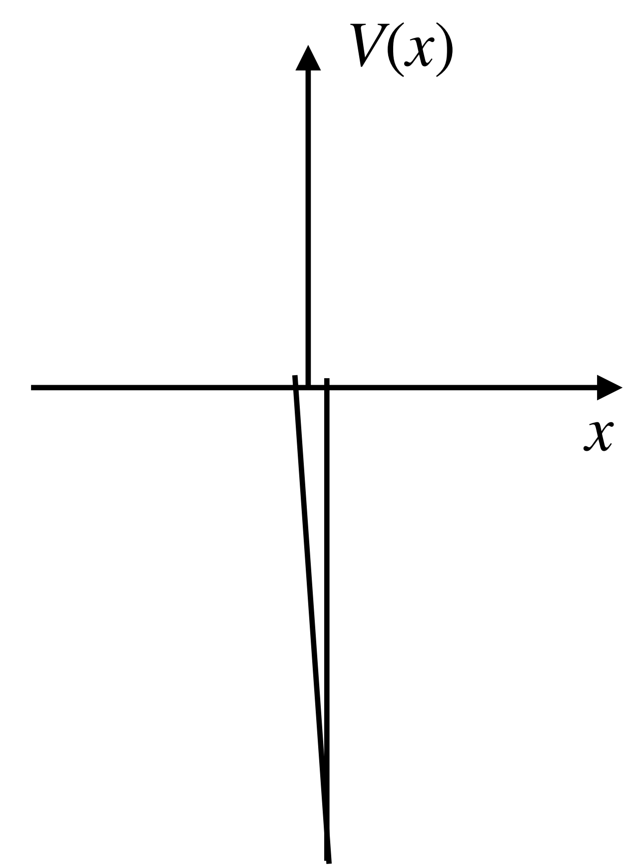
$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & \textbf{else} \end{cases}$$

The total area under it is 1. Note:

$$f(x)\delta(x-a) = f(a)\delta(x-a)$$



Suppose $V(x) = -\alpha\delta(x)$ with $\alpha > 0$

The Schrodinger equation gives

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \alpha\delta(x)\psi(x) = E\psi(x)$$

a. Bound states $E < V(\pm\infty) = 0$

b. Scattering states $E > 0$