Today

I. Zoom Tour
II. Last Time
III. Scattering on Delta Well

I. Last time

* Ethan showed us how to get a matrix from an operator

-Use an orthonormal basis $Q_{nm} = \langle n | \hat{Q} | m \rangle$

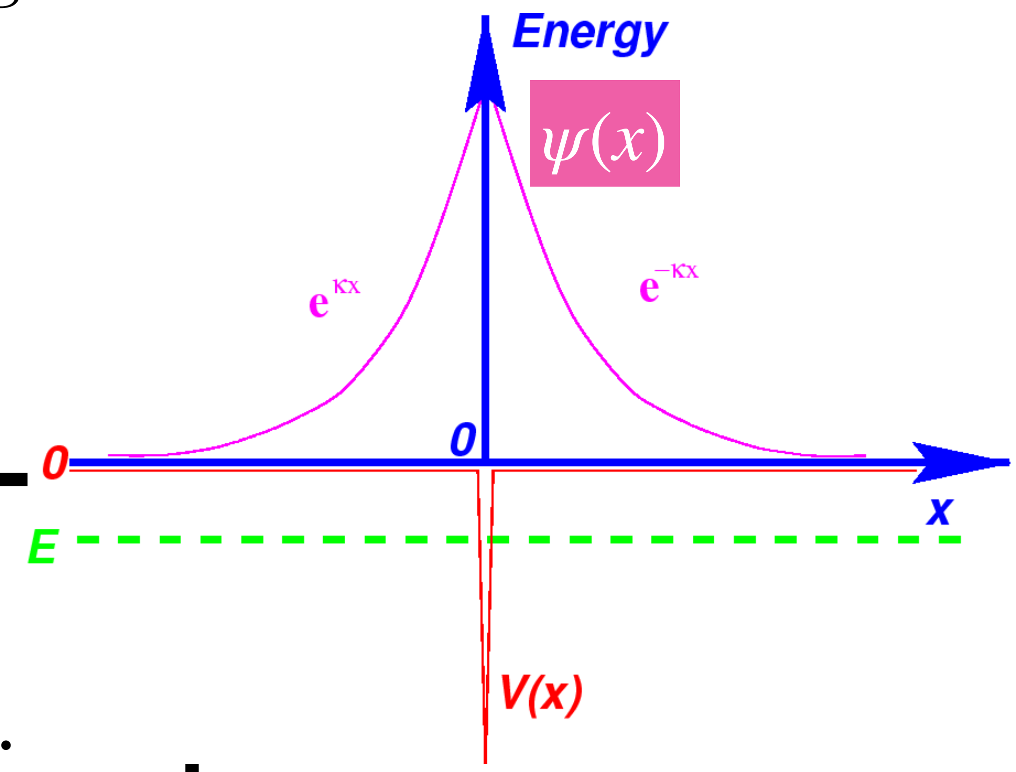
  * States are represented by vectors $|v\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
  * Operators act on states by matrix multiplication
  * To find eigenvalues we write down a characteristic equation:

$$\det(Q_{nm} - \lambda I_{nm}) = 0$$
$$\hat{Q}|v\rangle = \lambda |v\rangle$$

$$E = -\frac{k^2\hbar^2}{2m} = -\frac{m\alpha^2}{2\hbar^2}$$

Finally we determine B by normalizing:

$$B = \sqrt{k}$$



$$\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-\frac{m\alpha}{\hbar^2}|x|}, \quad \text{and} \quad E = -\frac{m\alpha^2}{2\hbar^2} \; .$$

Scattering states of the delta well with $E > 0$:

Wave function is continuous;

Derivative is continuous except when V is infinite.

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar}\psi = -k^2\psi$$

Solutions: $\psi(x) = Ae^{ikx} + Be^{-ikx} \qquad x < 0$

$\psi(x) = Fe^{ikx} + Ge^{-ikx} \qquad x > 0$

Continuity gives us $A + B = F + G$

$$\frac{d\psi}{dx} = Aike^{ikx} - Bike^{-ikx} \quad x < 0 \quad \text{and} \quad \frac{d\psi}{dx} = Fike^{ikx} - Gike^{-ikx} \quad x > 0$$

The discontinuity is

$$\Delta\frac{d\psi}{dx} = ik(F - G) - ik(A - B)$$

But also, it's supposed to be: $\Delta\frac{d\psi}{dx} = -\frac{2m\alpha}{\hbar^2}\psi(0)$

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$$\Delta \frac{d\psi}{dx} = ik(F - G) - ik(A - B)$$

But also, it's supposed to be: $\Delta \frac{d\psi}{dx} = -\frac{2m\alpha}{\hbar^2}\psi(0) = -\frac{2m\alpha}{\hbar^2}(A + B)$

We learn, after algebra that

$F - G = A(1 + 2i\beta) - B(1 - 2i\beta)$, where $\beta = \frac{m\alpha}{\hbar^2 k}$

For scattering:

A send a wave in $\longrightarrow$

B scattered wave $\longleftarrow$

F transmitted wave $\longrightarrow$

G send wave in $\longleftarrow$ (not done $G = 0$)

So, $G = 0$, now we can solve for B and F…

Continuity gives us $A + B = F$

$F = A(1 + 2i\beta) - B(1 - 2i\beta)$, where $\beta = \dfrac{m\alpha}{\hbar^2 k}$

$$B = \frac{i\beta}{1 - i\beta}A \quad \text{and} \quad F = \frac{1}{1 - i\beta}A$$

The relative probability of reflection:

$R = \dfrac{|B|^2}{|A|^2} = \dfrac{\beta^2}{1 + \beta^2}$ "reflection coefficient"

Tells you the fraction of particles that bounce back.

Similarly,

$T = \dfrac{|F|^2}{|A|^2} = \dfrac{1}{1 + \beta^2}$ "transmission coefficient"

Note: $R + T = 1$.✓

In terms of E:

$$R = \frac{1}{1 + \left(\frac{2\hbar^2 E}{m\alpha^2}\right)}$$

$$T = \frac{1}{1 + \left(\frac{\hbar^2 E}{2m\alpha^2}\right)}$$

How do we interpret these results? The wave functions are not normalizable. You proceed as before with wave packets—easy idea, But difficult in practice and is often implemented on a computer.

What if we did a δ-barrier? $\alpha \mapsto -\alpha$ and the R and T are the same As for the well!

The particle has a non-zero probability of going through an ∞ barrier!

It doesn't go over the barrier—instead it tunnels through. This makes All sorts of electronics, microscopy, and other experiments possible. (e.g. Josephson junctions, Scanning tunneling microscope (STM), etc.)