Today

I. Last Time

II. Complete the 'Old' Quantum Theory of Hydrogen

III. Antu's Guest Lecture: The Power Series Method for the Harmonic Oscillator

IV. Harmonic Oscillator Wave Functions

V. Power Series Method for Hydrogen

I. Last time

*Started to model the Hydrogen atom: electrostatic interaction plus circular orbits.

*Derived the radial equation—> got an effective potential

$$V_{\text{eff}} = V(r) + \frac{\hbar^2 \ell(\ell + 1)}{2mr^2}$$

I. Completing Our Discussion of the Radial Equation

This means that

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{2mr^2}{\hbar^2}[V(r) - E] = \ell(\ell + 1);$$

let's study this equation. Let's change variables to

$u(r) = rR(r)$ or $R(r) = u/r$. Let's compute derivatives

$$\frac{dR}{dr} = \frac{r\frac{du}{dr} - u}{r^2},$$

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = r^2\frac{d^2u}{dr^2}.$$

Then our differential equation in terms of u becomes

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}\right]u = Eu$$

Or

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V_{\text{eff}}u = Eu, \quad \text{where} \quad V_{\text{eff}}(r) = V(r) + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}$$

II. Completing Our Discussion of the Radial Equation

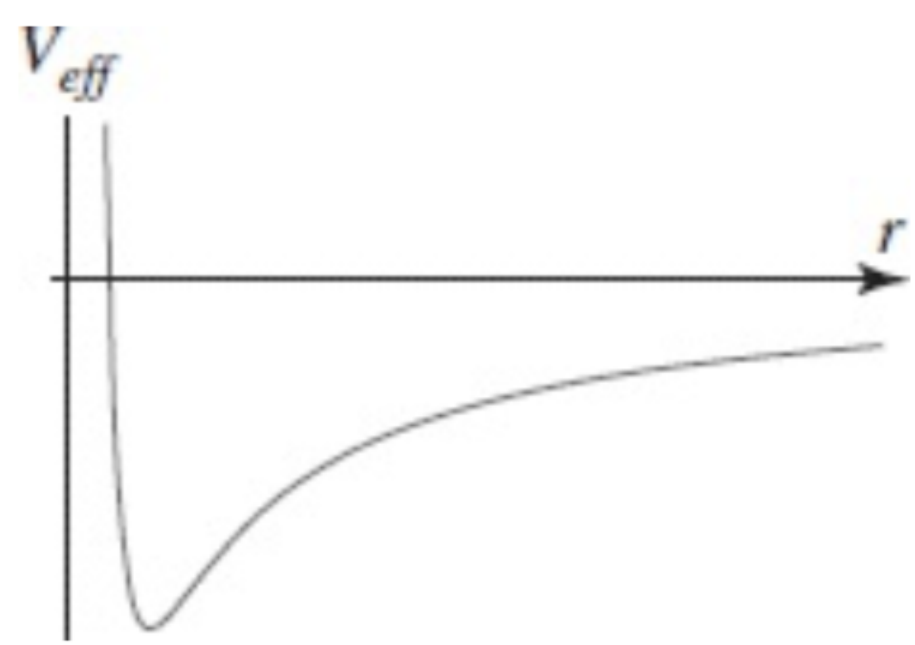
$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V_{\text{eff}}u = Eu, \quad \text{where} \quad V_{\text{eff}}(r) = V(r) + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}$$

Let's consider a particular potential, just to get a feel for what this looks like. The hydrogen atom potential is

$$V(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -\frac{k_e e^2}{r},$$

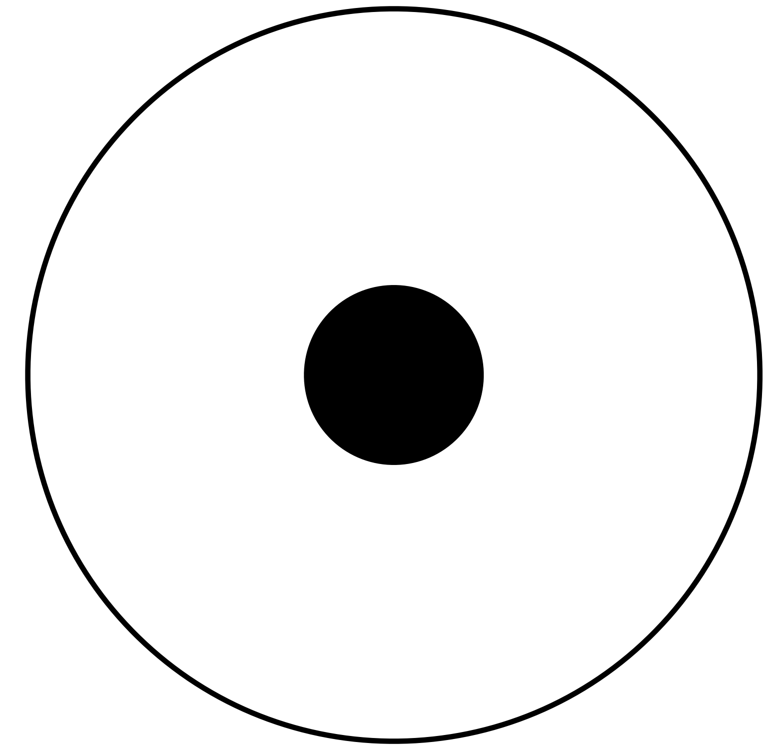
then the effective potential is

$$V_{\text{eff}} = -\frac{k_e e^2}{r} + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}$$



I. The 'Old' Quantum Theory of Hydrogen (The Bohr Model)

Classical Circular Orbit



$$\frac{mv^2}{r} = \frac{k_e e^2}{r^2}$$ centripetal force for a circular orbit

This gives

$$\frac{k_e e^2}{r} = mv^2.$$

The total energy in the system (KE+PE),

$$E = \left(\frac{1}{2}mv^2 - \frac{k_e e^2}{r}\right) = -\frac{1}{2}mv^2 = -\frac{1}{2}\frac{k_e e^2}{r}$$

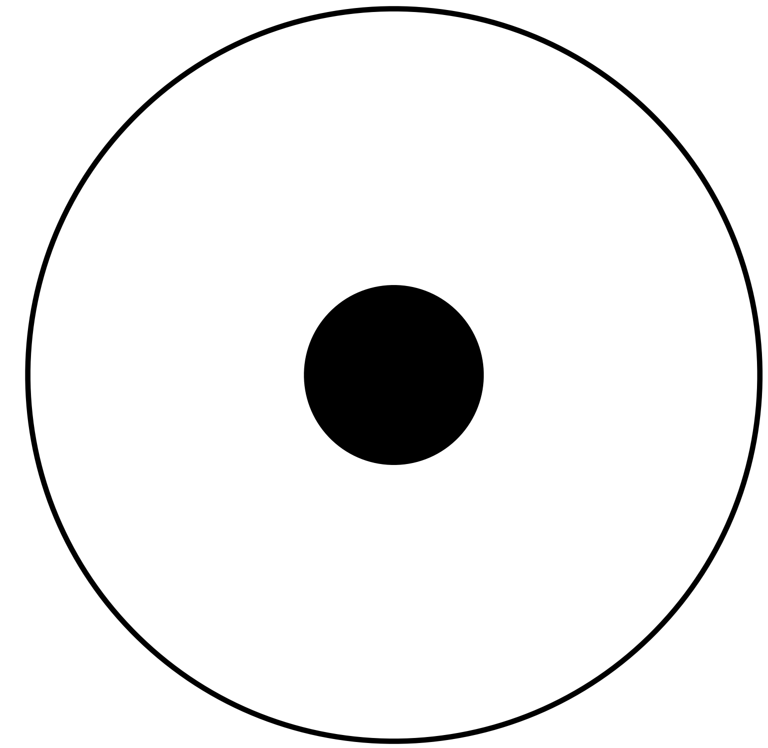
The angular momentum of this circular orbit is

$$|\overrightarrow{L}| = |\vec{r} \times \overrightarrow{p}| = rmv,$$

And

$$E = \left(-\frac{L^2}{2mr^2}\right) = -\frac{1}{2}\frac{k_e e^2}{r}.$$

II. The ‘Old’ Quantum Theory of Hydrogen (The Bohr Model)



The classical mechanics gave us:

$$E = \left(-\frac{L^2}{2mr^2} \right) = -\frac{1}{2}\frac{k_e e^2}{r}.$$

Bohr quantization: $L_n = n\hbar$,

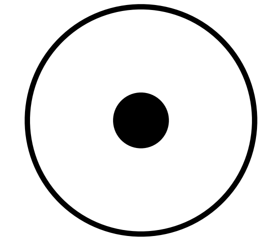
$$\frac{k_e e^2}{r_n} = \frac{(n\hbar)^2}{mr_n^2}.$$

Solve this for the radius to find $r_n = \left(\frac{\hbar^2}{k_e e^2 m} \right) n^2 = an^2$,

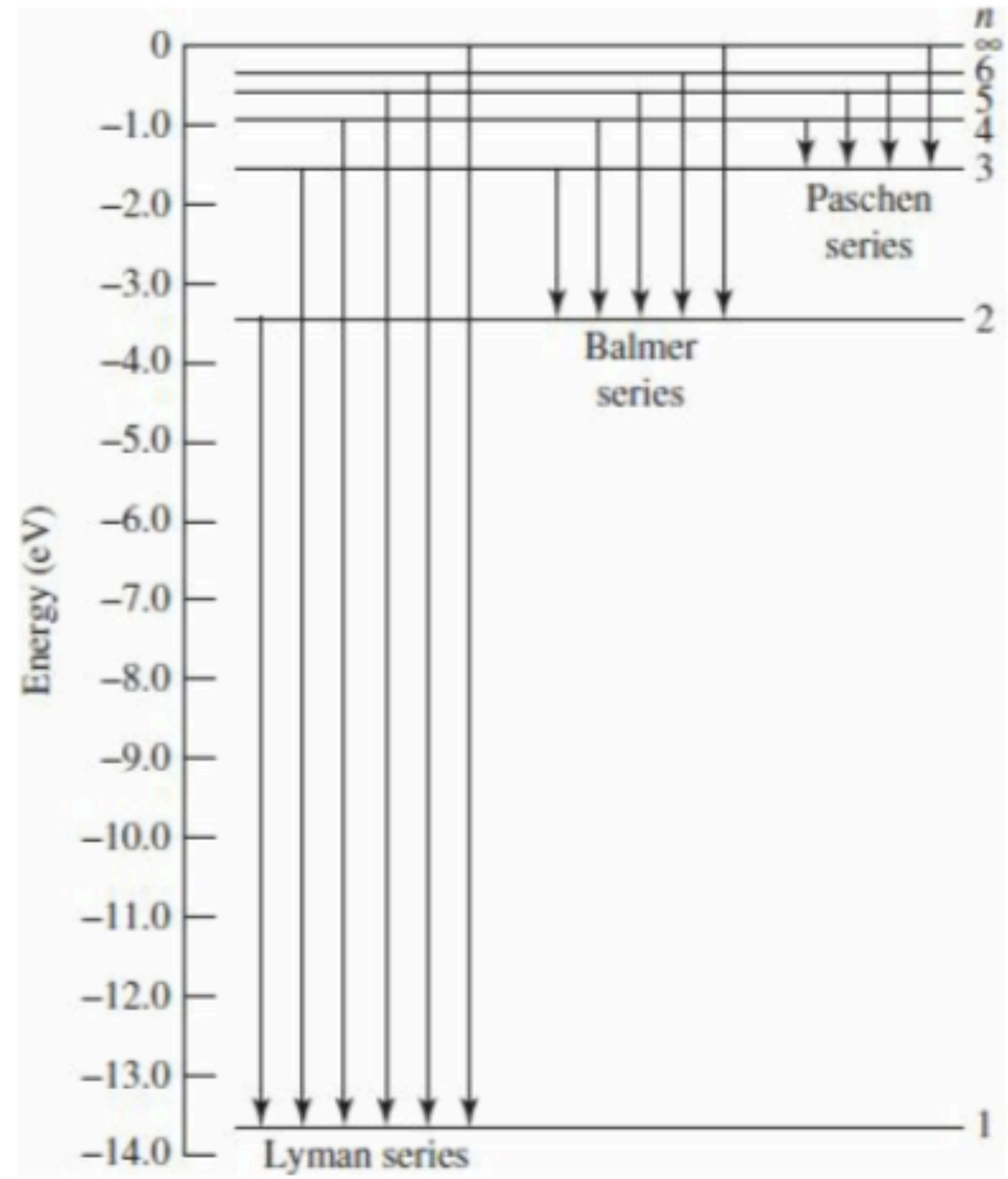
where a is the Bohr radius and has the value 0.529*10^(-10) m. This also leads to

$$E_n = \left(-\frac{1}{2}\frac{k_e e^2}{r_n} \right) = -E_g\frac{1}{n^2}, \text{ where } E_g = \frac{1}{2}\frac{k_e e^2}{a} = 13.6\text{eV}.$$

II. The 'Old' Quantum Theory of Hydrogen (The Bohr Model)



In quantum theory the energy eigenstates are stationary states and the Hydrogen atom is not accelerated in the classical sense. This is what allows the atom to be stable and not radiate.

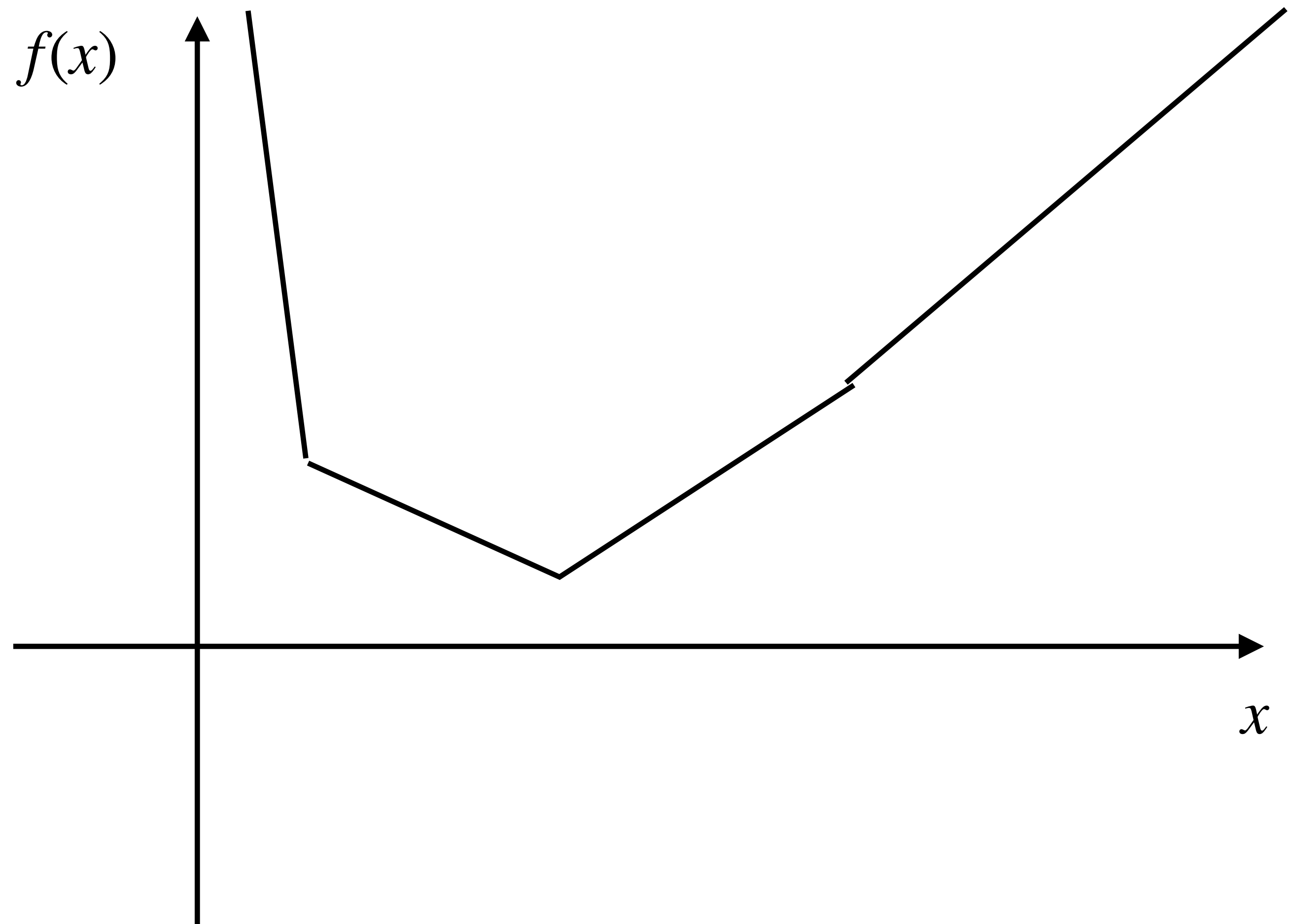


On the other hand, if we add energy into the system we can excite the electron to a higher energy state and it can radiatively decay to lower energy states.

$$E_\gamma = h\nu = |E_{n_i} - E_{n_f}|$$

II. Intuition for making sketches and the power series method

$$f(x) = \frac{1}{x^2} + x$$



When we find the asymptotic form for the solution to a differential equation, we're doing the same thing of filling in the boundaries that we need to interpolate between.