Homework 1

Due Friday, February 5th at 5pm

Read Chapters 1, 2 and 3 of Hartle's *Gravity*. Begin reading chapter 4 for next week.

1. What is the area of a circle of radius r (distance from center to circumference) in the two-dimensional geometry given by the surface of a sphere of radius a? Show that this reduces to πr^2 when $r \ll a$.

2. Draw examples of a triangle on the surface of a sphere for which:

   (a) The sum of interior angles is just slightly greater than π.

   (b) The sum of angles is equal to 2π.

   (c) What is the maximum the sum of angles of a triangle on a sphere can be according to the "spherical excess" formula we discussed in class? Can you exhibit a triangle where the sum achieves this value?

3. Consider the following coordinate transformation from familiar rectangular coordinates (x, y), labeling points in the plane to a new set of coordinates (μ, ν):

$$x = \mu\nu, \qquad y = \frac{1}{2}(\mu^2 - \nu^2)$$

   (a) Sketch, or use *Mathematica* to plot, the curves of constant μ and constant ν in the xy plane.

   (b) Transform the line element $dS^2 = dx^2 + dy^2$ into (μ, ν) coordinates.

   (c) Do the curves of constant μ and constant ν intersect at right angles?

   (d) Find the equation of a circle of radius r centered at the origin in terms of μ and ν.

   (e) Calculate the ratio of the circumference to the diameter of a circle using (μ, ν) coordinates. Do you get the correct answer?

4. *Equal-Area Projections* An equal-area map projection is one for which there is a constant proportionality between areas on the map and areas on the surface of the globe. Given $x = L\phi/2\pi$, what function $y(\lambda)$ would make an equal-area map? [Hint: If an infinitesimal area $dxdy$ has the same constant of proportionality to the corresponding infinitesimal area on the sphere wherever it is located, bigger areas will be also proportional.]

5. Show that Newton's laws of motion are *not* invariant under a transformation to a frame that is uniformly accelerated with respect to an inertial frame of Newtonian mechanics. What are the equations of motion in the accelerated frame?