Homework 2

Due Friday, February 12th at 5pm

Read Chapters 3 and 4 of Hartle's *Gravity*. Begin reading chapter 5 for next week.

1. Consider the functional
$$S[x(t)] = \int_0^T \left[\left(\frac{dx}{dt} \right)^2 + x^2 \right] dt.$$
Find the curve $x(t)$ satisfying the conditions $x(0) = 0$ and $x(T) = 1$, which makes $S[x(t)]$ an extremum. What is the extremum value of $S[x(t)]$? Is it a maximum or minimum?

2. The goal of this problem is to find the geodesics on the cylinder. Use cylindrical coordinates (ρ, ϕ, z), here ρ and ϕ are the standard polar coordinates in the x-y plane and z is the coordinate for the third, vertical direction.

   (a) Fix a cylinder of radius $\rho = R$. What is the line element on the surface of this cylinder?

   (b) Fix two points on the surface of the cylinder, say (R, ϕ_1, z_1) and (R, ϕ_2, z_2). Setup the integral that gives the arc length of a path connecting these two points. [So that we all do this the same way, go ahead and take z to be the independent variable, so that your path is described by a function $\phi(z)$.]

   (c) Find and solve the Euler-Lagrange equations that guarantee the extremization of this integral.

   (d) To check your answer, imagine slitting the cylinder along a vertical line and opening it up to a flat plane. Show that your solution curves from (c) go into the geodesics of the flat plane by doing this.

3. According to clocks on the ground, two streetlights A and B situated 4 km apart were turned on precisely at 8:00 pm EST:

   (a) Which one turned on first according to passengers on a high-speed train moving from A straight toward B at a speed of $3/5c$?

   (b) How much later (in seconds) did the other light turn on?

   (c) In the frame of the earth, are the events corresponding to the lights turning on space-like, light-like, or time-like separated?

   (d) How about in the frame of the train?

4. Harlte 4.3, p73

5. Consider twins, Joe and Ed. Joe goes off in a straight line traveling at a speed of $\frac{24}{25}c$, for 7 years as measured on his clock, then reverses and returns at half the speed. Ed remains at home. Make a spacetime diagram showing the motion of Joe and Ed from Ed's point of view. When they return, what is the difference in ages between Joe and Ed?