Homework 5

Due Friday, March 4th at 5pm

Read Chapters 6 and 7 of Hartle's *Gravity*.

1. Two particle calculations: (a) Pions play an important role in the nuclei of atoms. In particular, a photon can combine with a proton to produce a neutron and a pion:

   $$\gamma + p \to n + \pi^+.$$

   Find the minimum energy (the threshold energy) a photon would have to have to produce a pion in this way in the frame in which the proton is at rest. Is this energy within reach of contemporary accelerators?
   (b) Particle A (energy E) hits particle B (at rest), producing particles $C_1, C_2, \ldots, C_n$: $A+B \to C_1 + C_2 + \cdots + C_n$. Calculate the threshold (i.e., minimum E) for this reaction, in terms of the various particle masses.

   $$\left[\text{Answer:} \quad E = \frac{M^2 - m_A^2 - m_B^2}{2m_B}, \qquad \text{where } M = m_1 + m_2 + \cdots + m_n.\right]$$

2. In the LEP particle accelerator at CERN, electrons and positrons travel in opposite directions around a circular ring approximately 10 km in radius at an energy of 100 GeV apiece.

   (a) How close are these particles to moving at the velocity of light?

   (b) Electrons and positrons can be stored for 2h. How many turns will an electron or positron make around the ring in this time?

3. Express the Einstein velocity addition rule in terms of the parameter θ used to describe Lorentz boosts in Hartle's Eq. (4.18). Can you give a geometric interpretation to your result?

4. *Aberration* Consider a star, which happens to be directly overhead (the zenith) at midnight in a direction that lies in the plane of the Earth's orbit. To observe the star through a telescope, the telescope axis must be tilted with respect to the zenith direction by a small angle in the direction the Earth is moving in its orbit. Explain why and calculate the angle. To simplify the situation you may assume that the Earth's orbit is approximately circular and, if necessary, that the rotation axis is perpendicular to the orbital plane.

5. A source and detector are spaced a certain angle apart on the edge of a rotating disk. The source emits radiation at a frequency ω_* in its instantaneous rest frame. What frequency is the radiation detected at? [Hint: Little information is given in this problem because little is needed.]