

Today

General Relativity

Feb. 1st, 2016 P1/3

Day 1

I What is G.R.?

II What is G.R.?

II Logistical Interlude  
III Principal of Equivalence  
Preview

(a) Generalized Special Relativity

(OK to work in any reference frame even acc.) (OK to use any ref. frame moving at constant velocity)

Come to class!

Ask questions!!

I'm indebted to my teachers:  
David Griffiths, Robert Littlejohn,  
Carlo Rovelli.

What is gravity?

(b) Einsteins theory of gravity

(i) Newtonian Gravity

Universal gravitation:  $\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$   
Second law:  $\vec{F} = m\vec{a}$

$m_2 \vec{a} = -\frac{Gm_1m_2}{r^2}\hat{r}$

But this law is inconsistent with Special Relativity (S.R.)

Leads to myriad wonderful questions:  
What is the nature of time?  
Space? What does it mean to  
make an observation? What is energy?

Contrast Elec. to Mag. — already  
relativistic  $\rightarrow$  Why not modify  
gravity just as we did with  
Coulomb's law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}?$$

(It doesn't work!) Here's why:

Inspiration: do for the law  
of universal gravitation what  
Maxwell, Ampere, Faraday did

for Coulomb's law (e.g.  $\vec{g} \rightarrow m$ ).  
[doesn't work]

Sources of E & B fields:  
electric charge & current

$$\rho, \vec{J} \Rightarrow \vec{J}^\mu = (c\rho, \vec{J})$$

While the fields  $\vec{E}$  &  $\vec{B}$  form a tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

kinetic, potential, & rest) via  $E = mc^2$ . So, maybe all forms of energy are sources of gravity.

Ok. But, energy is not a Lorentz scalar — it's one component of the energy-momentum 4-vector:

$$p^\mu = (E/c, \vec{p})$$

So, momentum will also be a source of gravity.

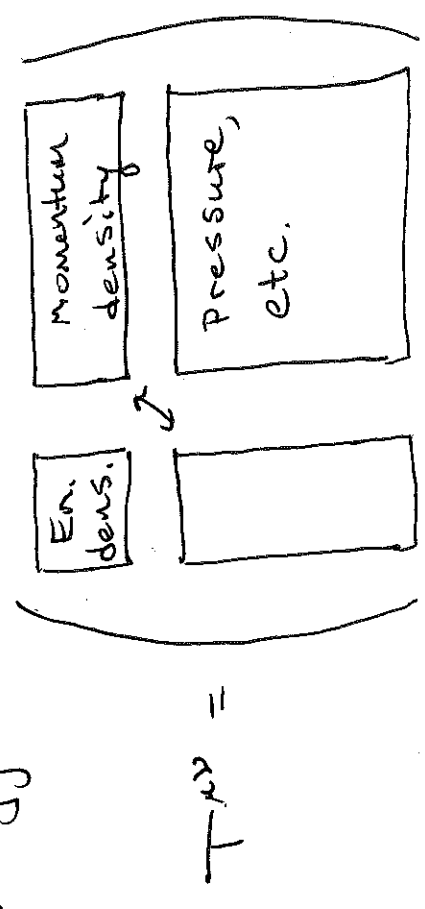
These are connected through Maxwell's equations:

$$\frac{\partial F^{\mu\nu}}{\partial x^\alpha} = \mu_0 J^\alpha$$

Why not substitute  $\rho$  (or  $\rho c$ )  $\rightarrow m$  (or  $\rho m$ )? Because mass, unlike charge, is not additive — total mass of composite structure reflects all forms of energy contained (e.g.

More over what we need is energy-momentum density — they fit together to make the

(energy-momentum-) stress tensor



## II Logistical Interlude

- (i) Schedule 4<sup>th</sup> class hour
- (ii) Schedule office hours

### Results:

- (i) Th 8-9pm
- (ii) Th 9-10pm and Fri 10-11am.

All objects fall under gravity with the same acceleration.

$$\text{Inertial mass} = \text{grav. mass}$$

This leads into the idea that gravity is geometry.

So we're looking for an equation of the form:

$$\boxed{???$$

source

constant containing Newton's constant  $G$ .

It took Einstein 12 years to find "??".

## III Principle of Equivalence Preview

Recall Newton:  $\vec{F} = -G \frac{Mm}{r^2} \hat{r}$

So, mass plays two unrelated roles — on the left it is inertia (i.e. a measure of  $|\vec{F}|/|a|$ ), and on the right the strength of gravity — and cancels out.