General Relativity

Day 12

Today

I Last time

II Light: Massless particles, energy, & momentum

III Doppler Shift

IV Introduce the equivalence principle.

I • Observers make local measurements by setting up axes and clocks:

$\underset{\sim}{e}_{\hat{0}} = \underset{\sim}{u}_{obs}, \quad \underset{\sim}{e}_{\hat{1}}, \underset{\sim}{e}_{\hat{2}}, \underset{\sim}{e}_{\hat{3}}$

• The energy and momentum that they measure is extracted using this frame

$$E = -\underset{\sim}{e}_{\hat{0}} \cdot \underset{\sim}{p} \qquad p^{\hat{i}} = \underset{\sim}{e}_{\hat{i}} \cdot \underset{\sim}{p}.$$

• Last night we introduced conservation of energy & momentum — in particular in the 4-form:

$$\boxed{\underset{\sim}{p}_{Ai} + \underset{\sim}{p}_{Bi} = \underset{\sim}{p}_{Af} + \underset{\sim}{p}_{Bf}}$$

II $m = 0$: In classical mechanics there's no such thing!

$p = mv = 0, \ T = \frac{1}{2}mv^2 = 0, \ F = ma = 0$

Forget it!

In relativity there's a loophole!

If $v = 1$ (or c) then the denominators are also zero:

$$E = \gamma m v = \frac{0}{0}? \qquad p = \gamma m v = \frac{0}{0}?$$

But,
$$E^2 - \vec{p}^2 = m^2 = 0$$
$$\Rightarrow \quad E^2 = \vec{p}^2$$
$$\Rightarrow \quad E = |\vec{p}|!$$

In fact, they <u>exist</u> (photons & gluons & gravitons!!).

This Experimental fact is a much stronger argument than the above loophole.

In the same year as relativity, Einstein recognized that for a photon,

$$E = \hbar\omega$$

$\hbar$ ← Planck's constant / 2π ≅ 10^{-34} J·s; ω ← angular frequency

The energy being $E = \hbar\omega$ and a piece of $\hbar$, motivates a convenient definition; we pull a factor of $\hbar$ out of $\vec{p}$ too:

$$\vec{p} = \hbar\vec{k}$$

and this implies (using $E = |\vec{p}|$)

$$|\vec{p}| = \hbar|\vec{k}| = E = \hbar\omega$$

or

$$\boxed{|\vec{k}| = \omega}$$

Also, because $\vec{k} \propto \vec{p}$ it points in the direction of motion.

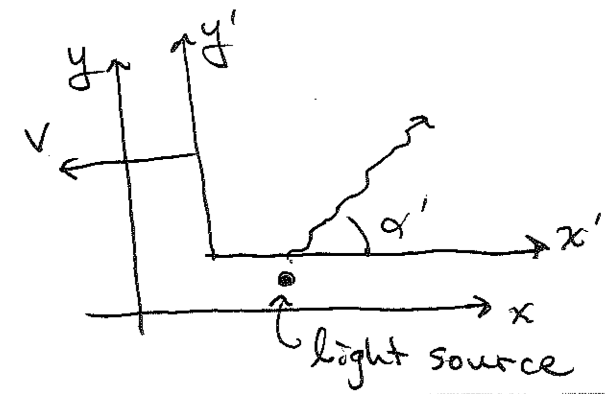
In summary, for light

$$\boxed{\text{light: } p^\alpha = (E, \vec{p}) = \hbar(\omega, \vec{k}) \equiv \hbar k^\alpha}$$

The 4-vector k^α is known as the wave 4-vector.

III Imagine a point source of light at rest in the lab (unprimed) frame. If it emits light of frequency ω in every direction, what does this look like in another frame?

Take the source to have a velocity $\vec{v}$ along the x'-axis of another frame, then this frame is moving in the lab frame as drawn



With this setup we know how $k^0 = \omega$ transforms,

$$k^0 = \omega = \gamma(\omega' - v k^{x'})$$

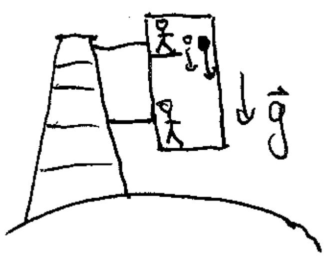
But, $k^{x'} = \omega' \cos\alpha'$, so,

$$\omega = \gamma(\omega' - v\omega' \cos\alpha')$$

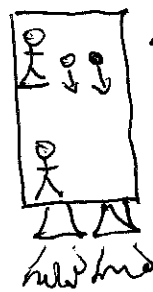
$$\Rightarrow \boxed{\omega' = \omega \frac{\sqrt{1 - v^2}}{1 - v\cos\alpha'}}$$

Doppler Effect.

IV Recall the parable of Galileo and the leaning tower of Pisa. This lead to Einstein's "happiest thought", a free fall observer has no idea there's a gravitational field. Conversely, an accelerated observer's experiments are identical to those of an observer immersed in a gravitational field pointing in the opposite direction:



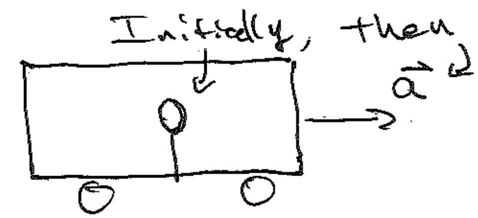
Same results as



$\uparrow \vec{a}$ with $|\vec{a}| = |\vec{g}|$.

Only an outside, inertial observer would say that these results happen for different reasons: gravitational attraction and rocket thruster respectively.

Example: Balloon in an accelerated train car



Which direction does the balloon deflect in?