General Relativity

Day 13

Feb 29th, 2016 P1/3

<u>Today</u>

I Last time

II Gravitational Time Dilation

[Note: Today we will once again use c, which will help us to keep track of the approximations we are making.]

I • Explored light:

$m_\gamma = 0$ & $E = \hbar\omega = |\vec{p}|$

Wrote,

$$\vec{p} = \hbar \vec{k} \quad w/ \quad |\vec{k}| = \omega$$

so that

$$p^\alpha = (E, \vec{p}) = \hbar(\omega, \vec{k}) = \hbar k^\alpha$$

• Derived the Doppler shift:

$$\omega' = \omega \frac{\sqrt{1 - v^2}}{1 - v\cos\alpha'}$$

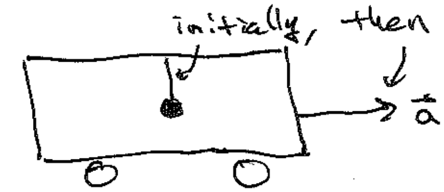
(check out relativistic beaming in text.)

• Discussed the relation

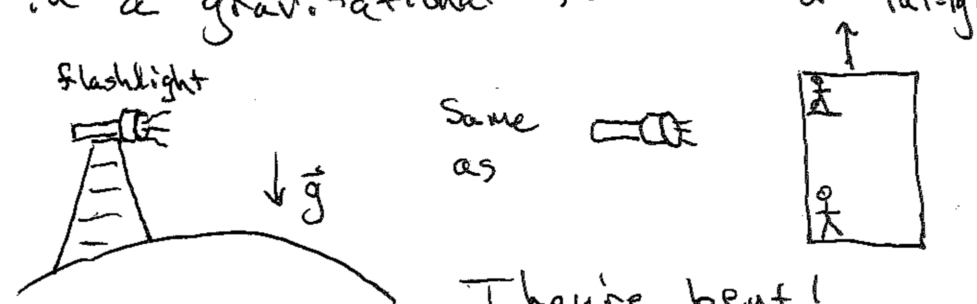
$$\boxed{m_I = m_G}$$

and Einstein's equivalence principle: a free fall observer has no idea there's a gravitational field.

Examples: How does a pendulum deflect in an accelerated train car?



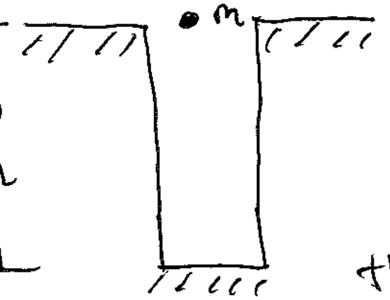
What happens to light rays in a gravitational field?



They're bent!

II "Naive" approach: Drop a rock down a mine shaft. Its kinetic energy is greater at the bottom,

$$E_2 = E_1 + mgh$$



What if I drop a photon?

If we use $m \to \frac{E}{c^2} = \frac{\hbar\omega}{c^2}$,

then $\hbar\omega_2 = \hbar\omega_1 + \frac{\hbar\omega_1}{c^2}\cdot gh$.

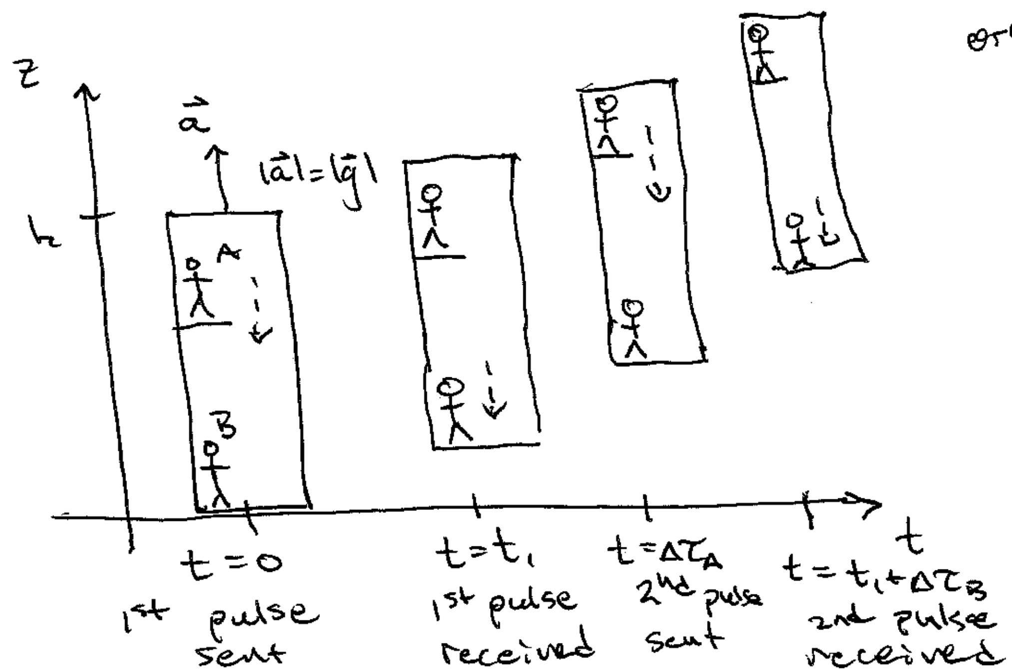
Simplifying gives

$$\boxed{\omega_2 = \omega_1\left(1 + \frac{gh}{c^2}\right)}$$

The photon is blue-shifted by the fall. This was confirmed in 1960 by Pound & Rebka. It is a tiny effect,

$$\frac{gh}{c^2} \approx \frac{10\,\mathrm{m/s^2}\,(20\,\mathrm{m})}{9\times10^{16}\,\mathrm{m^2/s^2}} \approx 2\times10^{-15}$$

Quantitative approach: (Hartle)



We will neglect $\left(\frac{v}{c}\right)^2$, $\left(\frac{gh}{c^2}\right)^2$, and higher orders, but keep v/c and $\frac{gh}{c^2}$. We have

$$z_B(t) = \tfrac{1}{2}gt^2$$

$$z_A(t) = h + \tfrac{1}{2}gt^2$$

The distance travelled by the 1st pulse is

$$z_A(0) - z_B(t_1) = ct_1$$

and that by the 2nd pulse is

$$z_A(\Delta\tau_A) - z_B(t_1+\Delta\tau_B) = c(t_1 + \Delta\tau_B - \Delta\tau_A).$$

If $\Delta\tau_A$ (hence $\Delta\tau_B$) is small then,

$$h - \frac{1}{2} g t_1^2 = c t_1 \qquad (1)$$

and

$$h - \frac{1}{2} g (t_1 + \Delta\tau_B)^2 \approx h - \frac{1}{2} g t_1^2 - g t_1 \Delta\tau_B$$
$$= c(t_1 + \Delta\tau_B - \Delta\tau_A) \qquad (2)$$

Computing Eq (1) - Eq (2) gives,

$$g t_1 \Delta\tau_B = c(\Delta\tau_A - \Delta\tau_B)$$

$$\Rightarrow \quad \Delta\tau_B = \frac{\Delta\tau_A}{(1 + g t_1 / c)}$$

If $\Delta\tau_A = 1\,\text{sec}$, then, because the denominator is positive and greater than 1, $\Delta\tau_B < 1\,\text{sec}$. Hence the lower clock runs slow. We can summarize this as:

"Slouching clocks run slow!"

But, from Eq. (1),

$$\frac{1}{2} g t_1^2 + c t_1 - h = 0$$

Quadratic Eq.

$$\Rightarrow t_1 = \frac{-c \pm \sqrt{c^2 + 2gh}}{g}$$
$$= -\frac{c}{g} \pm \frac{c}{g}\left(1 + \frac{gh}{c^2} + \dots\right)$$
$$= h/c$$

Thus,

$$\boxed{\Delta\tau_B = \frac{\Delta\tau_A}{\left(1 + \frac{gh}{c^2}\right)}}$$