General Relativity
Day 5

Feb. 10th, 2016

<u>Today</u>

I Geometry of Special Relativity (SR)

II Overview of relativistic effects

(A) Observer on the train:

Events R & F are simultaneous.

(B) observer on the ground:

R is before F.

Call this "the relativity of simultaneity".

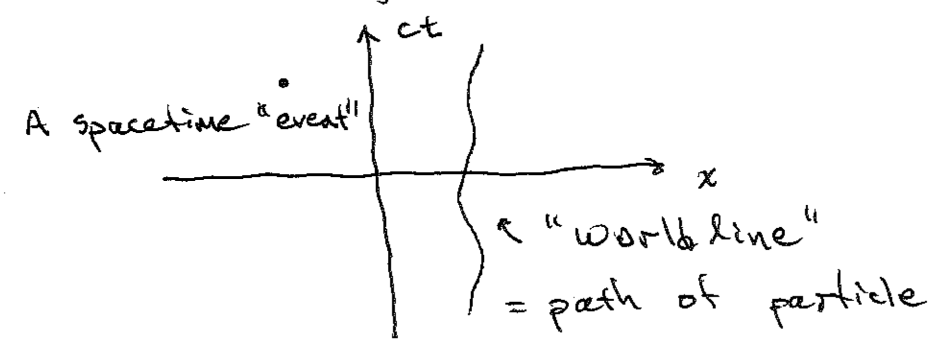
Last week we decided we would study the <u>geometry</u> of space and time.

We do this locally, so far just for space,

I. It is useful to add a definition to what we discussed last time: An <u>event</u> happens at a particular location at a particular time. In our train example, the detectors clicking are events

using
dS^2 = Squared distance btwn nearby pts.

Following Einstein, we would like to bring time into the picture and consider Spacetime:



What is the geometry of spacetime?
That is, what is ds^2?

Strategy: Turn our earlier observation about coordinates around:

$$\boxed{\text{Seek a distance all observers agree upon; an } \underline{\text{invariant}} \text{ distance.}}$$

In particular, it should be invariant under constant speed (S.R.) changes of frame.

In (t, x, y, z) coord.s

$$\Delta t = \frac{2L}{c} \qquad \Delta x = \Delta y = \Delta z = 0$$

while in (t', x', y', z') coord.s

$$\Delta t' = \frac{2}{c}\sqrt{L^2 + \left(\frac{\Delta x'}{2}\right)^2}, \quad \Delta x' = V\Delta t'$$

and $\Delta y' = \Delta z' = 0$.

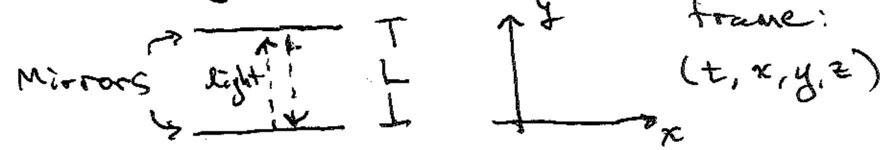
Spectacularly, these frames agree that

$$-(c\Delta t')^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2 = -4\left[L^2 + \frac{\Delta x'^2}{4}\right] + \Delta x'^2$$

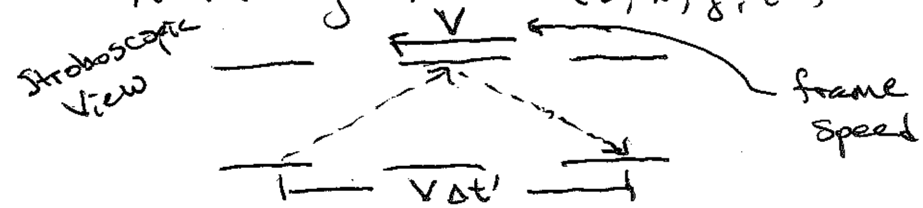
$$= -4L^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

We have already seen that different observers don't agree on time intervals ⟹ perhaps we have to mix up space & time.

<u>Motivating Example</u>: A light clock



A moving frame (t', x', y', z') sees



In fact, it is always true that

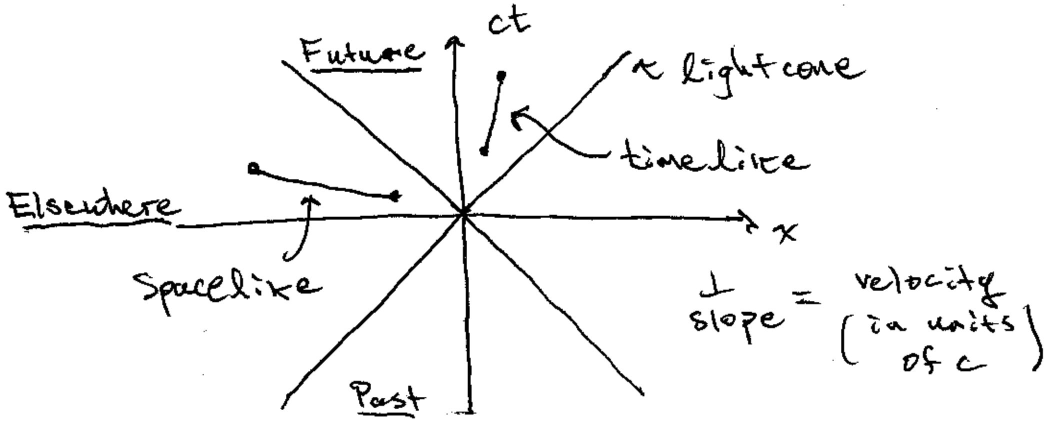
$$\Delta s^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$
$$= \text{Same primed}$$

in a flat spacetime. This provides an <u>invariant</u> generalization of $ds^2 = dx^2 + dy^2 + dz^2$:

$$\boxed{ds^2 = -c\,dt^2 + dx^2 + dy^2 + dz^2.}$$

Minkowski (= flat) spacetime line element.

Geometrically, all the intriguing effects of SR are due to the (surprising) $-c^2dt^2$.



$(\Delta s)^2 > 0$ Spacelike separated events

$(\Delta s)^2 = 0$ null or lightlike "

$(\Delta s)^2 < 0$ time like "

II Special relativistic effects

(You will derive these from scratch.)

(i) Lorentz Contraction

Moving rods are contracted (shorter)

length $L = L_*/\gamma$ ← rest length $\gamma \equiv \dfrac{1}{\sqrt{1 - V^2/c^2}}$

in moving frame

$\boxed{\begin{array}{c}\gamma \geq 1\\ \text{always}\end{array}}$

(ii) Time dilation

Moving clocks run slow

$$dt = \gamma\, d\tau$$

Proper Time:

Wall clock

"Proper time" Measured on particle's own watch, τ

Ordinary "coordinate-time", t

We will show briefly that

$$d\tau^2 = -ds^2/c^2$$

Thus, for timelike curves = world-lines, proper time is a nice, Lorentz-invariant, measure of "distance" along the curve.

(iii) Velocity addition

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}} \quad \left(\begin{array}{c}\text{along}\\ \text{boost}\end{array}\right)$$

More specifically,

$$V^{x'} = \frac{V^x - v}{1 - vV^x/c^2}\,;\quad V^{y'} = \frac{V^y/\gamma}{1 - vV^x/c^2}\,;\quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$V^{z'} = \frac{V^z/\gamma}{\quad}$$

All three effects (i), (ii), and (iii) follow from the relativity of simultaneity!