A DIAGONAL METRIC WORKSHEET

Consider the following general diagonal metric:

$$ds^2 = -A(dx^0)^2 + B(dx^1)^2 + C(dx^2)^2 + D(dx^3)^2$$

where dx^0, dx^1, dx^2, and dx^3 are completely arbitrary coordinates and A, B, C, and D are arbitrary functions of any or all of the coordinates. This worksheet (adapted from results listed in Rindler, *Essential Relativity*, 2/e, Springer-Verlag, 1977) allows you to quickly calculate the components of $\Gamma^{\alpha}_{\mu\nu}$ and $R_{\mu\nu} \equiv +R^{\alpha}{}_{\mu\alpha\nu}$ for any specific special case of such a metric. In this worksheet, I use the following shorthand notation:

$$A_0 \equiv \frac{\partial A}{\partial x^0}, \quad B_{12} \equiv \frac{\partial^2 B}{\partial x^1 \partial x^2}, \text{ and so on.}$$

To use this worksheet, start by crossing out each tabulated term that is zero for the specific metric in question. For the remaining terms, write the term's value in the space above that term. For the Ricci tensor components, you can then gather the terms in the space provided at the bottom. To adapt this worksheet to smaller dimensional spaces or spacetimes, treat the metric components corresponding to any nonexistent coordinates as if they had the value 1 and the remaining metric components as being independent of the nonexistent coordinates.

CHRISTOFFEL SYMBOLS

$\Gamma^0_{00} = \frac{1}{2A}A_0$ $\quad\Gamma^0_{10} = \Gamma^0_{01} = \frac{1}{2A}A_1$ $\quad\Gamma^0_{20} = \Gamma^0_{02} = \frac{1}{2A}A_2$ $\quad\Gamma^0_{30} = \Gamma^0_{03} = \frac{1}{2A}A_3$

$\Gamma^0_{11} = \frac{1}{2A}B_0$ $\quad\Gamma^0_{22} = \frac{1}{2A}C_0$ $\quad\Gamma^0_{33} = \frac{1}{2A}D_0$ $\quad$ other $\Gamma^0_{\mu\nu} = 0$

$\Gamma^1_{01} = \Gamma^1_{10} = \frac{1}{2B}B_0$ $\quad\Gamma^1_{11} = \frac{1}{2B}B_1$ $\quad\Gamma^1_{12} = \Gamma^1_{21} = \frac{1}{2B}B_2$ $\quad\Gamma^1_{13} = \Gamma^1_{31} = \frac{1}{2B}B_3$

$\Gamma^1_{00} = \frac{1}{2B}A_1$ $\quad\Gamma^1_{22} = -\frac{1}{2B}C_1$ $\quad\Gamma^1_{33} = -\frac{1}{2B}D_1$ $\quad$ other $\Gamma^1_{\mu\nu} = 0$

$\Gamma^2_{02} = \Gamma^2_{20} = \frac{1}{2C}C_0$ $\quad\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2C}C_1$ $\quad\Gamma^2_{22} = \frac{1}{2C}C_2$ $\quad\Gamma^2_{32} = \Gamma^2_{23} = \frac{1}{2C}C_3$

$\Gamma^2_{00} = \frac{1}{2C}A_2$ $\quad\Gamma^2_{11} = -\frac{1}{2C}B_2$ $\quad\Gamma^2_{33} = -\frac{1}{2C}D_2$ $\quad$ other $\Gamma^2_{\mu\nu} = 0$

$\Gamma^3_{03} = \Gamma^3_{30} = \frac{1}{2D}D_0$ $\quad\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{2D}D_1$ $\quad\Gamma^3_{23} = \Gamma^3_{32} = \frac{1}{2D}D_2$ $\quad\Gamma^3_{33} = \frac{1}{2D}D_3$

$\Gamma^3_{00} = \frac{1}{2D}A_3$ $\quad\Gamma^3_{11} = -\frac{1}{2D}B_3$ $\quad\Gamma^3_{22} = -\frac{1}{2D}C_3$ $\quad$ other $\Gamma^3_{\mu\nu} = 0$

RICCI TENSOR COMPONENTS (following three pages)

$$
\begin{array}{llllllll}
R_{00} = & 0 & + & \tfrac{1}{2B}A_{11} & + & \tfrac{1}{2C}A_{22} & + & \tfrac{1}{2D}A_{33} \\
+ & 0 & - & \tfrac{1}{2B}B_{00} & - & \tfrac{1}{2C}C_{00} & - & \tfrac{1}{2D}D_{00} \\
+ & 0 & + & \tfrac{1}{4B^2}B_0^2 & + & \tfrac{1}{4C^2}C_0^2 & + & \tfrac{1}{4D^2}D_0^2 \\
+ & 0 & + & \tfrac{1}{4AB}A_0B_0 & + & \tfrac{1}{4AC}A_0C_0 & + & \tfrac{1}{4AD}A_0D_0 \\
- & \tfrac{1}{4BA}A_1A_1 & - & \tfrac{1}{4B^2}A_1B_1 & + & \tfrac{1}{4BC}A_1C_1 & + & \tfrac{1}{4BD}A_1D_1 \\
- & \tfrac{1}{4CA}A_2A_2 & + & \tfrac{1}{4CB}A_2B_2 & - & \tfrac{1}{4C^2}A_2C_2 & + & \tfrac{1}{4CD}A_2D_2 \\
- & \tfrac{1}{4DA}A_3A_3 & + & \tfrac{1}{4DB}A_3B_3 & + & \tfrac{1}{4DC}A_3C_3 & - & \tfrac{1}{4D^2}A_3D_3
\end{array}
$$

$R_{00} =$

$$
\begin{array}{llllllll}
R_{11} = & \tfrac{1}{2A}B_{00} & + & 0 & - & \tfrac{1}{2C}B_{22} & - & \tfrac{1}{2D}B_{33} \\
- & \tfrac{1}{2A}A_{11} & + & 0 & - & \tfrac{1}{2C}C_{11} & - & \tfrac{1}{2D}D_{11} \\
+ & \tfrac{1}{4A^2}A_1^2 & + & 0 & + & \tfrac{1}{4C^2}C_1^2 & + & \tfrac{1}{4D^2}D_1^2 \\
- & \tfrac{1}{4A^2}B_0A_0 & - & \tfrac{1}{4AB}B_0B_0 & + & \tfrac{1}{4AC}B_0C_0 & + & \tfrac{1}{4AD}B_0D_0 \\
+ & \tfrac{1}{4BA}B_1A_1 & + & 0 & + & \tfrac{1}{4BC}B_1C_1 & + & \tfrac{1}{4BD}B_1D_1 \\
- & \tfrac{1}{4CA}B_2A_2 & + & \tfrac{1}{4CB}B_2B_2 & + & \tfrac{1}{4C^2}B_2C_2 & - & \tfrac{1}{4CD}B_2D_2 \\
- & \tfrac{1}{4DA}B_3A_3 & + & \tfrac{1}{4DB}B_3B_3 & - & \tfrac{1}{4DC}B_3C_3 & + & \tfrac{1}{4D^2}B_3D_3
\end{array}
$$

$R_{11} =$

$$
\begin{aligned}
R_{22} = {} & \tfrac{1}{2A} C_{00} && - \tfrac{1}{2B} C_{11} && + 0 && - \tfrac{1}{2D} C_{33} \\
& - \tfrac{1}{2A} A_{22} && - \tfrac{1}{2B} B_{22} && + 0 && - \tfrac{1}{2D} D_{22} \\
& + \tfrac{1}{4A^2} A_2^2 && + \tfrac{1}{4B^2} B_2^2 && + 0 && + \tfrac{1}{4D^2} D_2^2 \\
& - \tfrac{1}{4A^2} C_0 A_0 && + \tfrac{1}{4AB} C_0 B_0 && - \tfrac{1}{4AC} C_0 C_0 && + \tfrac{1}{4AD} C_0 D_0 \\
& - \tfrac{1}{4BA} C_1 A_1 && + \tfrac{1}{4B^2} C_1 B_1 && + \tfrac{1}{4BC} C_1 C_1 && - \tfrac{1}{4BD} C_1 D_1 \\
& + \tfrac{1}{4CA} C_2 A_2 && + \tfrac{1}{4CB} C_2 B_2 && + 0 && + \tfrac{1}{4CD} C_2 D_2 \\
& - \tfrac{1}{4DA} C_3 A_3 && - \tfrac{1}{4DB} C_3 B_3 && + \tfrac{1}{4DC} C_3 C_3 && + \tfrac{1}{4D^2} C_3 D_3
\end{aligned}
$$

$R_{22} =$

$$
\begin{aligned}
R_{33} = {} & \tfrac{1}{2A} D_{00} && - \tfrac{1}{2B} D_{11} && - \tfrac{1}{2C} D_{22} && + 0 \\
& - \tfrac{1}{2A} A_{33} && - \tfrac{1}{2B} B_{33} && - \tfrac{1}{2C} C_{33} && + 0 \\
& + \tfrac{1}{4A^2} A_3^2 && + \tfrac{1}{4B^2} B_3^2 && + \tfrac{1}{4C^2} C_3^2 && + 0 \\
& - \tfrac{1}{4A^2} D_0 A_0 && + \tfrac{1}{4AB} D_0 B_0 && + \tfrac{1}{4AC} D_0 C_0 && - \tfrac{1}{4AD} D_0 D_0 \\
& - \tfrac{1}{4BA} D_1 A_1 && + \tfrac{1}{4B^2} D_1 B_1 && - \tfrac{1}{4BC} D_1 C_1 && + \tfrac{1}{4BD} D_1 D_1 \\
& - \tfrac{1}{4CA} D_2 A_2 && - \tfrac{1}{4CB} D_2 B_2 && + \tfrac{1}{4C^2} D_2 C_2 && + \tfrac{1}{4CD} D_2 D_2 \\
& + \tfrac{1}{4DA} D_3 A_3 && + \tfrac{1}{4DB} D_3 B_3 && + \tfrac{1}{4DC} D_3 C_3 && + 0
\end{aligned}
$$

$R_{33} =$

$$R_{01} = -\frac{1}{2C} C_{01} - \frac{1}{2D} D_{01} + \frac{1}{4C^2} C_0 C_1 + \frac{1}{4D^2} D_0 D_1$$

$$+ \frac{1}{4AC} A_1 C_0 + \frac{1}{4AD} A_1 D_0 + \frac{1}{4BC} B_0 C_1 + \frac{1}{4BD} B_0 D_1$$

$R_{01} =$

$$R_{02} = -\frac{1}{2B} B_{02} - \frac{1}{2D} D_{02} + \frac{1}{4B^2} B_0 B_2 + \frac{1}{4D^2} D_0 D_2$$

$$+ \frac{1}{4AB} A_2 B_0 + \frac{1}{4AD} A_2 D_0 + \frac{1}{4CB} C_0 B_2 + \frac{1}{4CD} C_0 D_2$$

$R_{02} =$

$$R_{03} = -\frac{1}{2B} B_{03} - \frac{1}{2C} C_{03} + \frac{1}{4B^2} B_0 B_3 + \frac{1}{4C^2} C_0 C_3$$

$$+ \frac{1}{4AB} A_3 B_0 + \frac{1}{4AC} A_3 C_0 + \frac{1}{4DB} D_0 B_3 + \frac{1}{4DC} D_0 C_3$$

$R_{03} =$

$$R_{12} = -\frac{1}{2A} A_{12} - \frac{1}{2D} D_{12} + \frac{1}{4A^2} A_1 A_2 + \frac{1}{4D^2} D_1 D_2$$

$$+ \frac{1}{4BA} B_2 A_1 + \frac{1}{4BD} B_2 D_1 + \frac{1}{4CA} C_1 A_2 + \frac{1}{4CD} C_1 D_2$$

$R_{12} =$

$$R_{13} = -\frac{1}{2A} A_{13} - \frac{1}{2C} C_{13} + \frac{1}{4A^2} A_1 A_3 + \frac{1}{4C^2} C_1 C_3$$

$$+ \frac{1}{4BA} B_3 A_1 + \frac{1}{4BC} B_3 C_1 + \frac{1}{4DA} D_1 A_3 + \frac{1}{4DC} D_1 C_3$$

$R_{13} =$

$$R_{23} = -\frac{1}{2A} A_{23} - \frac{1}{2B} B_{23} + \frac{1}{4A^2} A_2 A_3 + \frac{1}{4B^2} B_2 B_3$$

$$+ \frac{1}{4CA} C_3 A_2 + \frac{1}{4CB} C_3 B_2 + \frac{1}{4DA} D_2 A_3 + \frac{1}{4DB} D_2 B_3$$

$R_{23} =$