# Problem Solving 10: Solving the Einstein Equation

The Einstein equation can be written in multiple different forms. The first form that we derived in class was

$$\boxed{R_{\alpha\beta} = \frac{8\pi G}{c^4}\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T^{\gamma}_{\gamma}\right).}$$

This can be the easiest form in which to solve the equations, especially in vacuum. For completeness, it can equivalently be written

$$\boxed{G_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta},}$$

where

$$\begin{aligned} G_{\alpha\beta} &= R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R, \\ R^{\alpha}{}_{\beta\gamma\delta} &= \frac{\partial\Gamma^{\alpha}{}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial\Gamma^{\alpha}{}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\alpha}{}_{\gamma\epsilon}\Gamma^{\epsilon}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\delta\epsilon}\Gamma^{\epsilon}{}_{\beta\gamma}, \\ R_{\alpha\beta} &= R^{\gamma}_{\alpha\gamma\beta}, \\ R &= R^{\alpha}_{\alpha}. \end{aligned}$$

1. Solve this equation for a vacuum spacetime that is spherically symmetric and static.