BLC 190
Homework 5

Name:_____

1. Given that $b > 0$, evaluate the following:

   (a) $\log_b(1)$

   (b) $\log_b(b^3)$

   (c) $\log_b(\sqrt{b})$

   (d) $\log_b\left(\dfrac{1}{b}\right)$

2. Solve the following equations:

   (a) $3^t = 81$

   (b) $5^x = 7$

   (c) $5e^{3t} = 10$

   (d) $\log_3(x - 1) = 2$

3. For each of the following equations, state whether the equation is true or false.

   (a) $\ln(x + y) = \ln x + \ln y$

   (c) $\ln(xy) = (\ln x)(\ln y)$

   (b) $\ln(xy) = \ln x + \ln y$

   (d) $e^x e^y = e^{x+y}$

4. Solve the following equations:

(a) $\log_4(x) + \log_4(x - 6) = 2$

(b) $4 = \dfrac{5}{1 + 3e^{-x}}$

(c) $e^{2x} - 2e^x - 8 = 0$

5. At noon, a bacteria culture contains 2000 bacteria. After three hours the population has increased to 7800.

(a) Assuming exponential growth, find a formula for the number of bacteria in the culture after t hours.

(b) How many bacteria are in the culture at 6:30pm?

(c) At what time does the culture contain 50,000 bacteria?

6. The half-life of thorium-229 is 7340 years. A sample contains 5 grams of thorium-229.

(a) Find a formula for the amount of thorium-229 in the sample after t years.

(b) How much thorium-229 is in the sample after 5000 years?

(c) How long will it take the sample to contain 2 grams of thorium-229?

7. The population of fish in a lake is modeled by the following equation:

$$P = \frac{5000}{1 + 3e^{-0.04t}}$$

where P is the number of fish in the lake after t years.

(a) How many fish are in the lake after 3 years?

(b) How many fish are in the lake after 7 years?

(c) When will there be 4000 fish in the lake?

(d) What is the eventual population of fish in the lake?

8. In chemistry, the pH of a solution is defined as:

$$pH = -\log_{10}\left[H^+\right]$$

where the symbol $\left[H^+\right]$ denotes the concentration of hydrogen ions in solution measured in moles per liter.

(a) Suppose that the concentration of hydrogen ions in a solution is 4×10^{-8} moles per liter. What is the pH of the solution?

(b) The pH of water is 7. Determine the concentration of hydrogen ions in water.