

Practice Problems for Final Exam

1. Solve the following equations:

(a) $x^2 - 5x + 6 = 0$

(b) $x^2 = 12 - x$

(c) $(x^2 - 4)(2x - 3) = 0$

(d) $x^3 + 13x^2 + 42x = 0$

(e) $\frac{1}{x} + \frac{3}{x^2} = 0$

(f) $\frac{x}{x+5} - \frac{1}{x-3} = 0$

(g) $3\sqrt{x} - 2 = \sqrt{x} + 6$

(h) $(\sqrt{x} - 1)^2 = x - 5$

(i) $x^{-2} = 4$

(j) $x^{2/3} = 16$

(k) $x^{-1/2} = \frac{1}{3}$

(l) $\frac{32}{\sqrt{x}} = \sqrt[3]{x}$

2. Simplify by combining the fractions:

(a) $\frac{1}{x} + \frac{x}{x+2}$

(b) $\frac{1}{x-3} + \frac{1}{(x-3)(x+2)}$

3. Simplify the following expressions:

(a) $\frac{\frac{1}{x} - \frac{2}{y}}{z}$

(b) $\frac{z}{\frac{3}{x} + \frac{1}{x^2}}$

4. Rationalize the denominators of the following fractions:

(a) $\frac{2}{3 - \sqrt{5}}$

(b) $\frac{1}{x - \sqrt{y}}$

5. Simplify the following expressions:

(a) $x(x^3y^2)^4$

(b) $\sqrt[3]{x^9y^6}$

(c) $\frac{(x^2y^4)^3}{x^{-2}y^5}$

(d) $\frac{x^{3/4}}{x^{1/2}}$

6. Find all solutions to the following system of equations:

$$\begin{aligned}x - 2y &= 7 \\5x + 3y &= -4\end{aligned}$$

7. Given that $b > 0$, evaluate the following:

(a) $\log_b(b^4)$

(b) $\log_b(1)$

(c) $\log_b(\sqrt[5]{b})$

(d) $\log_b\left(\frac{1}{\sqrt{b}}\right)$

8. Solve the following equations:

(a) $2^{3x-4} = 10$

(b) $e^{3x} = 7$

(c) $\ln(x-1) = 5$

(d) $\log_2(x) + \log_2(x-4) = 5$

(e) $\frac{3}{5-2e^{-x}} = 4$

(f) $e^{2x} - 7e^x + 10 = 0$

9. Let $f(x) = 3x + \frac{1}{x}$ and $g(x) = \sqrt{x}$.

(a) What is $f(g(x))$?

(b) What is $g(f(x))$?

10. Find the equations for the lines through the following points.

(a) $(1, -1)$ and $(3, 5)$

(b) $(-1, 0)$ and $(-2, 5)$

(c) $(-1, 5)$ and $(2, 5)$

(d) $(-3, 2)$ and $(-3, 5)$

11. Consider the line $3x + 5y = 2$.

(a) What is the slope of this line?

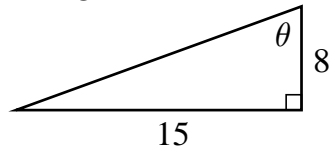
(b) What is the y -intercept of this line?

(c) What is the x -intercept of this line?

12. Convert $\frac{5\pi}{6}$ radians to degrees.

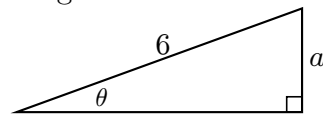
13. Convert 150° to radians.

14. Consider the following right triangle:



What is $\sin \theta$?

15. Consider the following right triangle:



If $\theta = 25^\circ$, what is a ?

16. Use the Unit Circle to evaluate the following trigonometric functions. Give exact answers.

(a) $\sin\left(\frac{\pi}{4}\right)$

(d) $\cos\left(\frac{7\pi}{6}\right)$

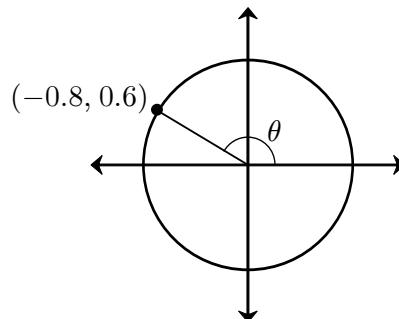
(b) $\tan\left(\frac{\pi}{3}\right)$

(e) $\csc\left(\frac{2\pi}{3}\right)$

(c) $\sin\left(\frac{7\pi}{3}\right)$

(f) $\cos\left(\frac{17\pi}{6}\right)$

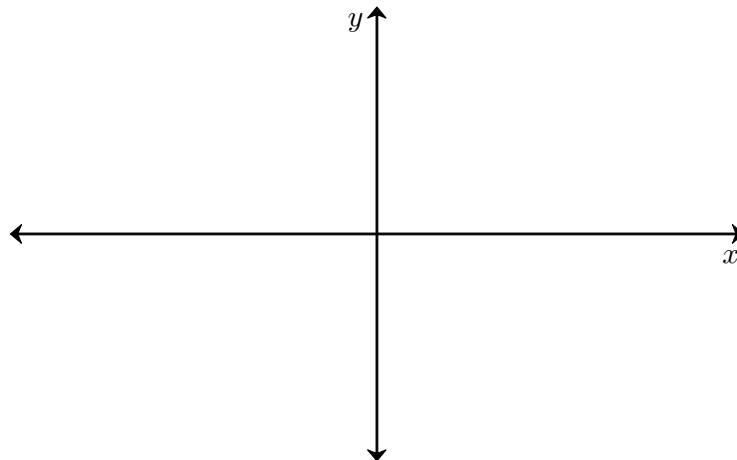
17. The following circle has radius 1 and is centered at the origin:



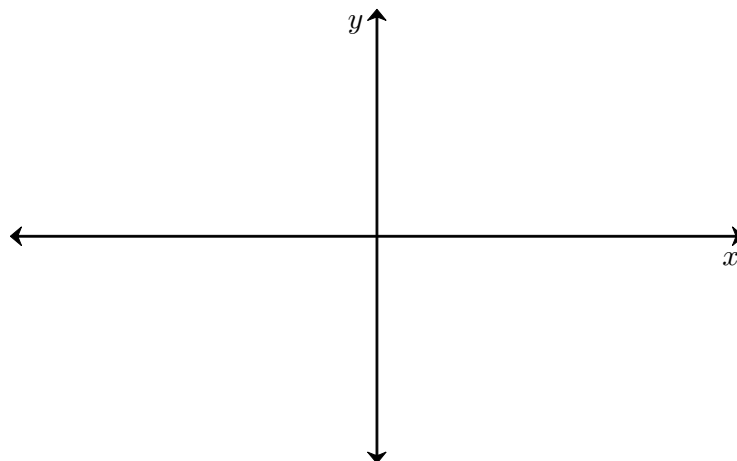
What is $\tan \theta$?

18. Sketch the graphs of the following functions:

(a) $y = 2 \sin x + 1$

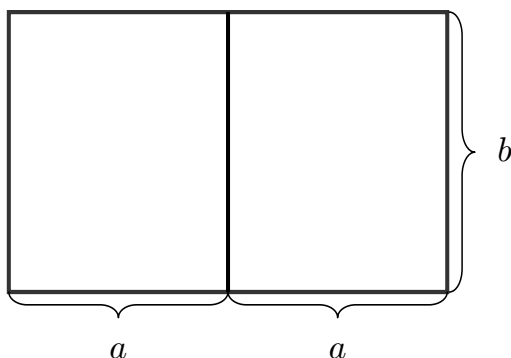


(b) $y = \cos(2x)$



19. At age 18, Megan owned 40 books. After age 18, she buys 4 books a year. How many books does she own at age x ?
20. In 2000, the population of Red Hook was 1805, and in 2002, the population was 1824. Assuming that the population grows linearly, estimate the population of Red Hook in 2008.
21. Women born in 1960 have a life expectancy of 73.1 years; women born in 2004 have a life expectancy of 80.4 years. For the following questions, assume that life expectancy is linearly related to year of birth.
- (a) Find an equation for the life expectancy of a woman born in year x .
- (b) What would you predict for the life expectancy of a woman born in 2009?

22. A rancher wishes to fence off a rectangular plot of land. The plot will be divided into two equal portions by an additional fence parallel to the two sides:



If the rancher has 9 km of fence, express the area of the region as a function of a .

23. At noon a bacteria culture contains 700 bacteria. At 2pm, the culture contains 1000 bacteria.
- (a) Assuming exponential growth, determine the number of bacteria in the culture at 4:30pm.
 - (b) At what time will the culture contain 2000 bacteria?
24. The half-life of plutonium-239 is 25,000 years. A sample initially contains 16 grams of plutonium-239.
- (a) How much will the sample contain after 40,000 years?
 - (b) When will the sample contain 14 grams of plutonium-239?
25. The spread of influenza in a certain city is modeled by the following equation:

$$P = \frac{500,000}{1 + 2499e^{-0.92t}}$$

where P is the number of people who have become ill with the flu t weeks after the initial outbreak.

- (a) How many people have become ill 5 weeks after the initial outbreak?
- (b) When will 100,000 have become ill?