

Exercises: Differentials

1. Suppose that $y = \sqrt{x}$.
- Find a formula for dy in terms of x and dx .
 - Use your formula to estimate the change in y if we increase x from 4.00 to 4.08.

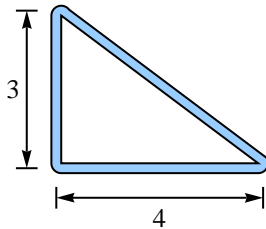
2. Suppose that $y = x^3 - x$.
- Find a formula for dy in terms of x and dx .
 - Given that $x = 2.000 \pm 0.003$, use differentials to estimate the range of possible values for y .

3. In physics, the kinetic energy of a moving object is given by the formula

$$K = \frac{1}{2}mv^2.$$

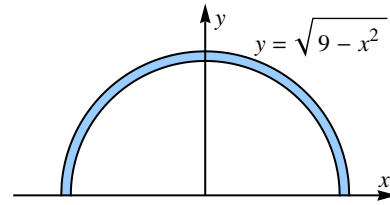
- Assuming m is constant, find a formula for dK in terms of m , v , and dv .
- A moving object with a mass of 2.44 kg is measured to have a velocity of 1.26 ± 0.02 m/s. Find the kinetic energy of the object, and use differentials to estimate the error in this value.

4. In the following picture, the shaded region has a uniform thickness of 0.02.



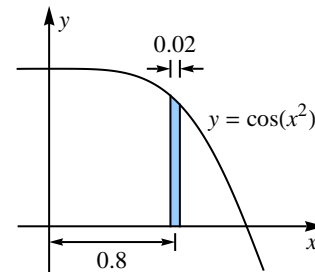
Use differentials to estimate the area of the shaded region.

5. In the following picture, the shaded region has a uniform thickness of 0.05.



Use differentials to estimate the area of the shaded region.

6. Use differentials to estimate the area of the shaded region in the following figure.



7. A ping-pong ball is made of plastic with a thickness of 0.4 mm. Given that the ball has a radius of 20 mm, estimate the total volume of plastic used to make the ball.

8. A plastic cup has the shape of a cylinder with an open top. The cup has a radius of 3.0 cm and a height of 9.0 cm, and the plastic has a thickness of 0.08 cm. Estimate the total volume of plastic used to make the cup.

Answers

1. (a) $dy = \frac{1}{2\sqrt{x}} dx$ (b) 0.02 **2.** (a) $dy = (3x^2 - 1) dx$ (b) 6.000 ± 0.033 **3.** (a) $dK = mv dv$ (b) $1.94 \pm 0.06 \text{ kg} \cdot \text{m}^2/\text{s}^2$

4. 0.24 **5.** $0.15\pi \approx 0.4712$ **6.** 0.016 **7.** $640\pi \text{ mm}^3 \approx 2010.6 \text{ mm}^3$ **8.** $5.04\pi \text{ cm}^3 \approx 15.83 \text{ cm}^3$