Math 141 Name: SOLUTION

Exam 1

1. **[8 points (4 pts each)]** The Mauna Loa Observatory observatory in Hawaii regularly measures the concentration of CO_2 in the atmosphere. In the year 2000, the observatory measured 371 ppm (parts per million) of CO_2 in the atmosphere; in 2010, the observatory measured 393 ppm of CO_2 in the atmosphere.

   (a) Use linear approximation to estimate the concentration of CO_2 in the atmosphere in 2015.

$$\frac{393-371}{2010-2000} = \frac{22}{10} = 2.2 \text{ ppm/year}$$

so $C \approx 393 + 2.2(t-2010)$

Check

$t = 2010$ gives $C = 393$

$t = 2000$ gives $C = 371$

Plug in $t = 2015$

$C \approx \boxed{404 \text{ ppm}}$

   (b) According to the linear approximation, in what year will atmospheric CO_2 reach a concentration of 440 ppm?

Plug in $C = 440$

$440 = 393 + 2.2(t-2010)$

$\Downarrow$ solve for t

$t = 2031.36$

$\boxed{2031 \text{ or } 2032}$

Check

2000: 371 ppm

2010: 393 ppm

2020: 415 ppm

2030: 437 ppm

2031: 439.2 ppm

440 ppm $\rightarrow$

2032: 441.4 ppm

2. **[15 points (5 pts each)]** Evaluate each of the following limits.

(a) $\lim_{x\to\infty} \frac{x^2}{x-4x^2} = \lim_{x\to\infty} \frac{x^2}{-4x^2} = \boxed{-\frac{1}{4}}$

Check

x	$\frac{x^2}{x-4x^2}$
100	-0.250627
1000	-0.250062
10,000	-0.250006
$\downarrow$	$\downarrow$
∞	-0.25

(b) $\lim_{x\to-3} \frac{x^2-9}{x+3} = \lim_{x\to-3} \frac{(x-3)(x+3)}{x+3} = \lim_{x\to-3} x-3 = \boxed{-6}$

Check

x	$\frac{x^2-9}{x+3}$
-2.9	-5.9
-2.99	-5.99
-2.999	-5.999
$\downarrow$	$\downarrow$
-3	-6

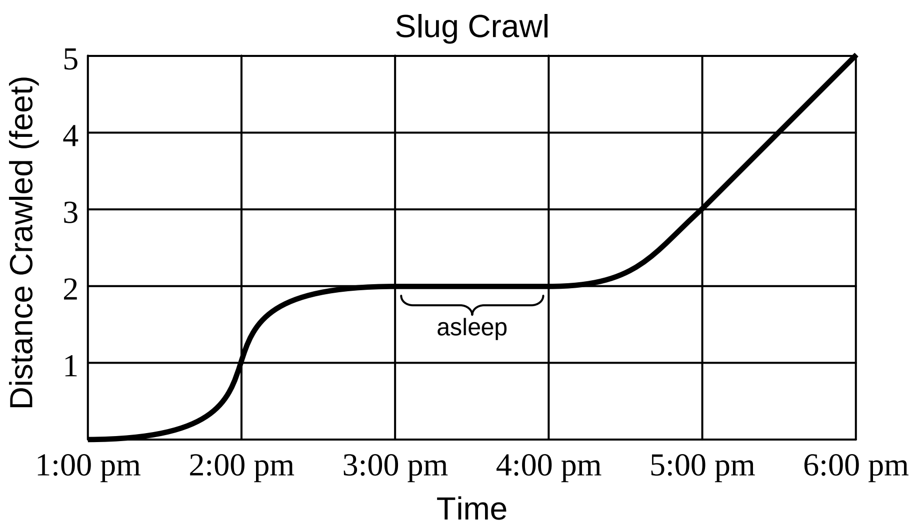
(c) $\lim_{h\to0} \frac{(3h+2)^2-4}{h}$

$$= \lim_{h\to0} \frac{9h^2+12h+4-4}{h} = \lim_{h\to0} 9h+12 = \boxed{12}$$

Check

h	$\frac{(3h+2)^2-4}{h}$
0.01	12.09
0.001	12.009
0.0001	12.0009
$\downarrow$	$\downarrow$
0	12

3. **[24 points]** A particularly fast-moving slug is crawling straight across an 8-foot-wide driveway. The following graph shows the distance crawled by the slug as a function of time:



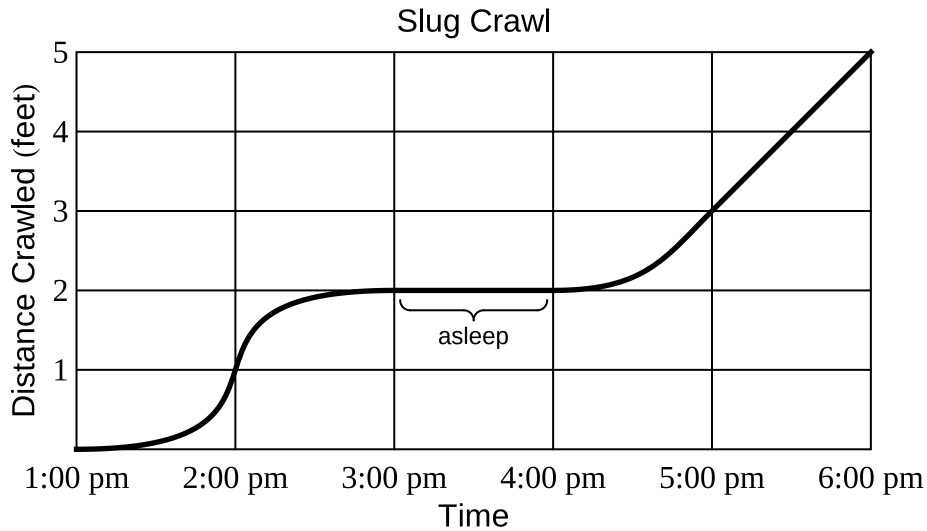
(a) **[5 pts]** What was the average speed of the slug between 1:00 pm and 6:00 pm? What was his average speed between 4:00 pm and 6:00 pm?

$$\frac{5-0}{6\text{pm}-1\text{pm}} = \frac{1 \text{ foot}}{1 \text{ hour}} = \boxed{1 \text{ foot}/\text{hour}}$$

$$\frac{5-2}{6\text{pm}-4\text{pm}} = \frac{3 \text{ feet}}{2 \text{ hours}} = \boxed{1.5 \text{ feet}/\text{hour}}$$

(b) **[5 pts]** At what time was the slug crawling the fastest? Explain.

The slug was crawling fastest at $\boxed{2{:}00 \text{ pm}}$, since the slope is the steepest at this time.



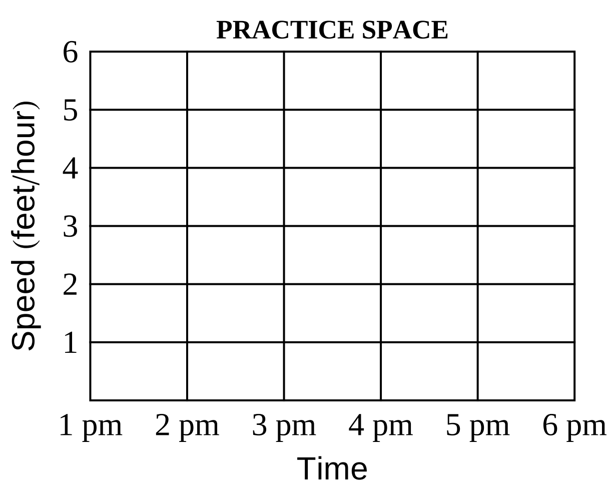
(c) **[8 pts]** What was the slug's crawl speed at 6:00 pm? Assuming he can maintain this pace, when will he reach the other end of the 8-foot-wide driveway?

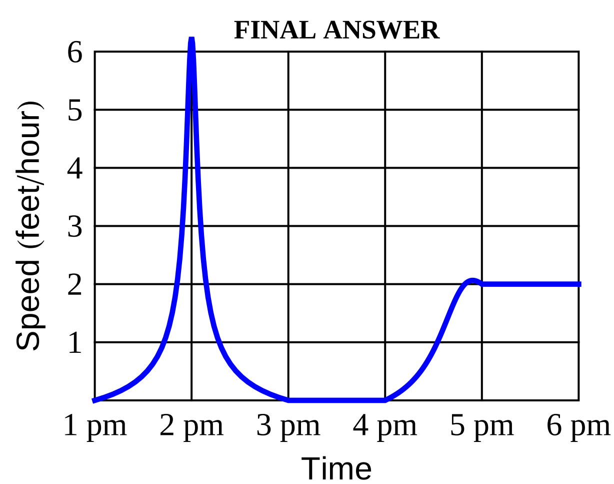
6 pm : $\boxed{\text{2 ft/hour}}$

3 more feet @ 2 ft/hour = $1\frac{1}{2}$ hours

$\boxed{\text{7:30 pm}}$

(d) **[6 pts]** Sketch a graph of the slug's speed as a function of time:





4. **[18 points (6 pts each)]**

(a) Find $f'(x)$ if $f(x) = 8x^2 + \dfrac{10}{\sqrt{x}} + 3.$

$$= 8x^2 + 10x^{-1/2} + 3$$

$$f'(x) = \boxed{16x - 5x^{-3/2}}$$

$$= 16x - \frac{5}{x\sqrt{x}}$$

(b) Find $f'(x)$ if $f(x) = \sqrt{1+x^2}.$

$$= (1+x^2)^{1/2}$$

$$f'(x) = \boxed{\frac{1}{2}(1+x^2)^{-1/2}(2x)}$$

$$= x(1+x^2)^{-1/2}$$

$$= \frac{x}{\sqrt{1+x^2}}$$

(c) Find $\dfrac{dy}{dx}$ if $y = x\left(x^2-1\right)^4.$

$$\frac{dy}{dx} = \boxed{(x^2-1)^4 + 4x(x^2-1)^3(2x)}$$

$$= (x^2-1)^4 + 8x^2(x^2-1)^3$$

$$= (x^2-1)(x^2-1)^3 + 8x^2(x^2-1)^3$$

$$= (9x^2-1)(x^2-1)^3$$

5. **[7 points]** The following table shows some values for a function f:

x	$f(x)$
4.9	9.559122451
4.99	8.150599012
4.999	8.015005999
4.9999	8.001500060
5	8

Use this data to estimate $f'(5)$. Your answer must be correct to within 0.01.

$$\frac{8 - f(4.9)}{5-4.9} = -15.591$$

$$\frac{8 - f(4.99)}{5-4.99} = -15.0599$$

$$\frac{8 - f(4.999)}{5-4.999} = -15.006$$

$$\frac{8 - f(4.9999)}{5-4.9999} = -15.0006$$

$$\boxed{-15}$$

6. **[8 points]** Let $f(x) = \dfrac{6x}{x^2-1}$. Find the equation of the tangent line to the graph of f at the point $(2,4)$.

$$f(x) = 6x(x^2-1)^{-1}$$

$$f'(x) = 6(x^2-1)^{-1} - 6x(x^2-1)^{-2}(2x)$$

$$f'(2) = 6(3)^{-1} - 6(2)(3)^{-2}(4) = -\frac{10}{3}$$

$$\boxed{y = 4 - \frac{10}{3}(x-2)}$$

7. **[12 points (6 pts each)]** In the theory of electrical circuits, ***Joule's law*** describes a relationship between the power P dissipated by a resistor, the electrical current I passing through the resistor, and the resistance R. The law can be written as follows:

$$P = I^2 R.$$

Usually power is measured in watts, current is measured in amps, and resistance is measured in ohms, where 1 ohm $=$ 1 watt/amp^2.

(a) Find an equation for $\frac{dP}{dt}$ in terms of I, R, $\frac{dI}{dt}$, and $\frac{dR}{dt}$.

$$\frac{dP}{dt} = R\left(2I\frac{dI}{dt}\right) + I^2\frac{dR}{dt}$$

(product rule)

(b) Suppose that the current through a resistor is increasing at a rate of 0.04 amps/sec, while the resistance is increasing at a rate of 5 ohms/sec. How quickly is the power dissipated by the resistor increasing when the current is 0.3 amps and the resistance is 60 ohms?

$I = 0.3$ amps $\qquad R = 60$ ohms

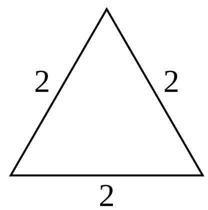
$\frac{dI}{dt} = 0.04$ amps/sec $\qquad \frac{dR}{dt} = 5$ ohms/sec

so $\frac{dP}{dt} = \boxed{1.89 \text{ watts/sec}}$

8. **[8 points]** Define a function f as follows:

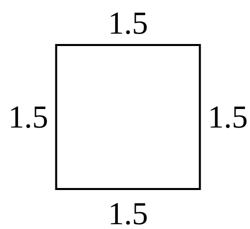
$$f(n) \;=\; \text{the area of a regular polygon with } n \text{ sides and perimeter 6.}$$

For example, $f(3)$ is the area of an equilateral triangle whose perimeter is 6:



$$f(3) \;=\; \text{the area of this triangle} \;\approx\; 1.732,$$

and $f(4)$ is the area of a square whose perimeter is 6:



$$f(4) \;=\; \text{the area of this square} \;=\; 2.25.$$

What is $\lim_{n\to\infty} f(n)$? Explain your answer.

As $n \to \infty$, the polygon looks more and more like a circle with perimeter 6.

$$2\pi r = 6$$

$$r = \frac{3}{\pi}$$

The radius of such a circle is $3/\pi$, so the area is $\pi r^2 = \pi\left(\frac{3}{\pi}\right)^2 = \boxed{\frac{9}{\pi}}$