

Exercises: Exponentials and Logarithms

1–8 ■ Find the exact value of each expression. Do not use a calculator.

1. $\log_{10}(1000)$
2. $\log_{10}(0.00001)$
3. $\log_{10}(1)$
4. $\log_{10}(\sqrt{10})$
5. $\ln(e^3)$
6. $\ln(1/e)$
7. $\ln(e\sqrt{e})$
8. $\ln(-e)$

9–14 ■ Solve for x . Round your answers to the nearest thousandth.

9. $5^x = 8$
10. $3e^{4x} = 16$
11. $\log_{10}(5x) = 2$
12. $3 \ln(2x + 1) = 8$
13. $\ln(x^3) = 4 + \ln(x^2)$
14. $\ln(5 + e^x) = 3$

15–16 ■ Express the given quantity as a single logarithm

15. $\log_{10}(5) + 2 \log_{10}(3)$
16. $\frac{\ln(36)}{2} - \ln(2)$

17–20 ■ Express the given quantity as a power of e .

17. $(e^x)^5$
18. $\frac{1}{e^x}$
19. 3^x
20. $x^{\sqrt{x}}$

21–30 ■ Compute $f'(x)$.

21. $f(x) = 5e^{3x} + 2 \ln(x)$
22. $f(x) = 4x^2 e^{-x}$
23. $f(x) = \ln(x^3 + 5x + 2)$
24. $f(x) = \ln(\cos x)$
25. $f(x) = \frac{e^{x/2}}{5}$
26. $f(x) = x^3 \ln(\sin x)$
27. $f(x) = \frac{1}{1 + e^{4x}}$
28. $f(x) = \ln(1 + e^{\sin x})$
29. $f(x) = 5^x$
30. $f(x) = x^{\sin x}$

31. Find the equation of the tangent line to the curve $y = 2e^{3x}$ at the point $(0, 2)$.

32. Find the equation of the tangent line to the curve $y = \ln x$ at the point $(5, \ln 5)$.

33–36 ■ Based on the given data, classify the function $f(x)$ as linear, exponential, or neither.

33. $f(0) = 4$, $f(1) = 6$, and $f(2) = 9$.
34. $f(0) = 6$, $f(10) = 8$, and $f(20) = 10$.
35. $f(0) = 80$, $f(0.1) = 60$, and $f(0.2) = 45$.
36. $f(0) = 10$, $f(1) = 20$, and $f(2) = 35$.

37–38 ■ Find the formula for an exponential function $f(x)$ satisfying the given conditions.

37. $f(0) = 0.4$, $f(2) = 1.8$
38. $f(0) = 8$, $f'(0) = 12$

39. At $t = 0$, a colony of bacteria has a population of 10,000. By $t = 20$ min, the population has increased to 18,000.

- (a) Assuming exponential growth, find a formula for the population of bacteria after t minutes.
- (b) When will the population reach 100,000?
- (c) How quickly will the population be growing at this time?

40. A 2.00-kg sample of radioactive ^{137}Cs is buried underground. Five years later, the sample is exhumed, and is found to have a mass of only 1.78 kg.

- (a) Assuming exponential decay, find a formula for the mass of the sample after t years.
- (b) What is the half-life of ^{137}Cs ?

41. A capacitor with an initial charge of 5.0 coulombs is attached to a lightbulb. At the moment that the circuit is connected, the capacitor begins to discharge at a rate of 0.08 coulombs/sec.

- (a) Assuming the charge decays exponentially, find a formula for the charge on the capacitor after t seconds.
- (b) How long will it take for the capacitor to discharge 90% of its initial charge?

42. A 0.2-mole sample of ^{155}Eu has been prepared for study. Based on Geiger counter readings, the sample is decaying at a rate of 0.00008 moles/day.

- (a) Find a formula for the amount of ^{155}Eu that will be left after t days.
- (b) What is the half-life of ^{155}Eu ? Express your answer in years.

Answers

1. 3 2. -5 3. 0 4. $1/2$ 5. 3 6. -1 7. 1.5 8. undefined 9. 1.292 10. 0.418 11. 20 12. 6.696

13. 54.598 14. 2.714 15. $\log_{10}(45)$ 16. $\ln(3)$ 17. e^{5x} 18. e^{-x} 19. $e^{x \ln(3)}$ 20. $e^{\sqrt{x} \ln(x)}$

21. $15e^{3x} + \frac{2}{x}$ 22. $(8x - 4x^2)e^{-x}$ 23. $\frac{3x^2 + 5}{x^3 + 5x + 2}$ 24. $-\tan x$ 25. $\frac{e^{x/2}}{10}$ 26. $3x^2 \ln(\sin x) + x^3 \cot x$

27. $-\frac{4e^{4x}}{(1 + e^{4x})^2}$ 28. $\frac{e^{\sin x} \cos x}{1 + e^{\sin x}}$ 29. $5^x \ln(5)$ 30. $\left(\frac{\sin x}{x} + \ln(x) \cos(x) \right) x^{\sin x}$

31. $y = 2 + 6x$ 32. $y = \ln(5) + \frac{1}{5}(x - 5)$ 33. exponential 34. linear 35. exponential 36. neither

37. $0.4e^{(0.752039)x}$ 38. $8e^{1.5x}$ 39. (a) $10,000e^{(0.029389)t}$ (b) $t = 78.3$ min (c) 2939 bacteria/min

40. (a) $2e^{(-0.023307)t}$ (b) 29.7 years 41. (a) $5e^{(-0.016)t}$ coulombs (b) 144 sec

42. (a) $0.2e^{(-0.0004)t}$ moles (b) 4.74 years