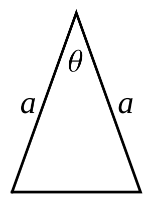
Math 141

Name:________________________________

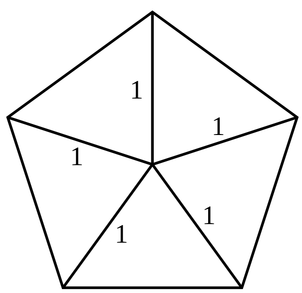
Homework 2

1. Consider the following formula for the area of an isosceles triangle:

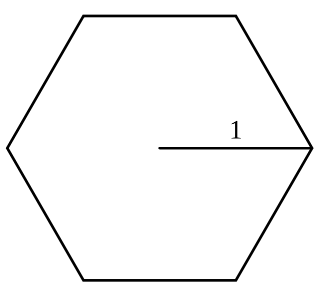


$$\text{Area} = \frac{1}{2} a^2 \sin \theta$$

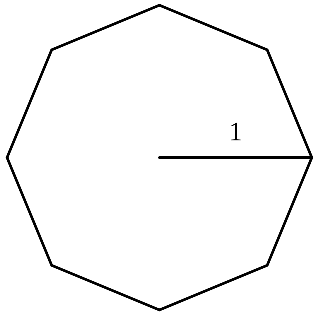
   (a) A **unit pentagon** is a regular pentagon with a radius of 1, as shown in the following figure. Use the area formula above to find the area of a unit pentagon.



   (b) Find the area of a unit hexagon. Express your answer as a decimal.



   (c) Find the area of a unit octagon. Express your answer as a decimal.



Let $f(n)$ be the area of a unit polygon with n sides.

(d) Find a formula for $f(n)$. Make sure to check that your formula is consistent with your answers to parts (a), (b), and (c).

(e) Make a table of values for f showing $f(10)$, $f(100)$, $f(1000)$, and $f(10{,}000)$. Based on your table, what is the value of $\lim_{n\to\infty} f(n)$?

(f) Give a brief geometric explanation for your answer to part (e).

2. Consider the following sum:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

The first term is $1/2$, and each successive term is half as big as the previous term. For example, $1/4$ is half of $1/2$, and $1/8$ is half of $1/4$. The sum goes on forever, with infinitely many terms.

(a) List the denominators of the first ten terms.

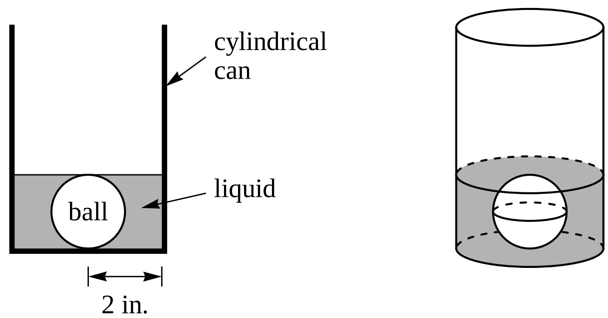
(b) Let $g(n)$ be the result of adding together the first n terms. For example,

$$g(1) = \frac{1}{2}, \qquad g(2) = \frac{1}{2} + \frac{1}{4}, \qquad \text{and} \qquad g(3) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}.$$

Make a table showing the value of $g(n)$ for $n = 1, 2, \ldots, 10$. Express your answers as decimals.

(c) Based on your answer to part (b), what would the result be of adding together *all* of the terms? Explain.

3. A heavy metal ball is placed into a tin can, and then liquid is added until the top of the ball is just barely covered:

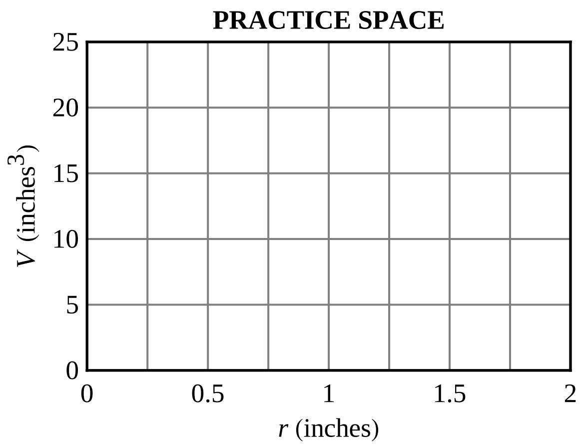


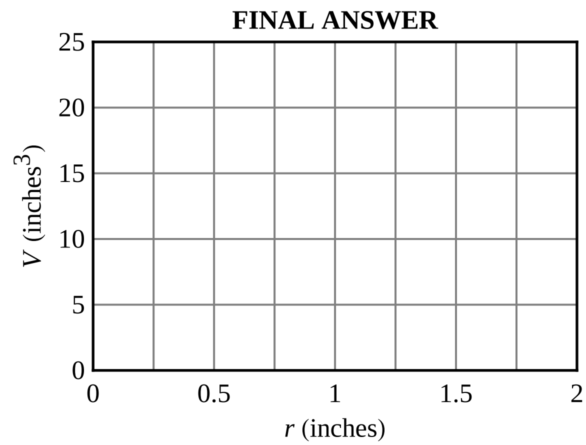
The can is a cylinder with a radius of 2 inches, and the ball is a sphere of radius r.

(a) Compute the volume of liquid in the can when r is 0.5 inches.

(b) Find a formula for the volume V of liquid in the can as a function of r. Make sure that your formula is consistent with your answer to part (a).

(c) Use the following axes to draw a careful graph of V as a function of r. Feel free to use a graphing calculator or computer to help you with this part.





(d) Explain geometrically why the volume V is initially increasing.

(e) Explain geometrically why the volume V decreases for larger values of r.

(f) Use a graphing calculator or computer to find the value of r that requires the maximum amount of liquid. Your answer should be correct to within 0.01.