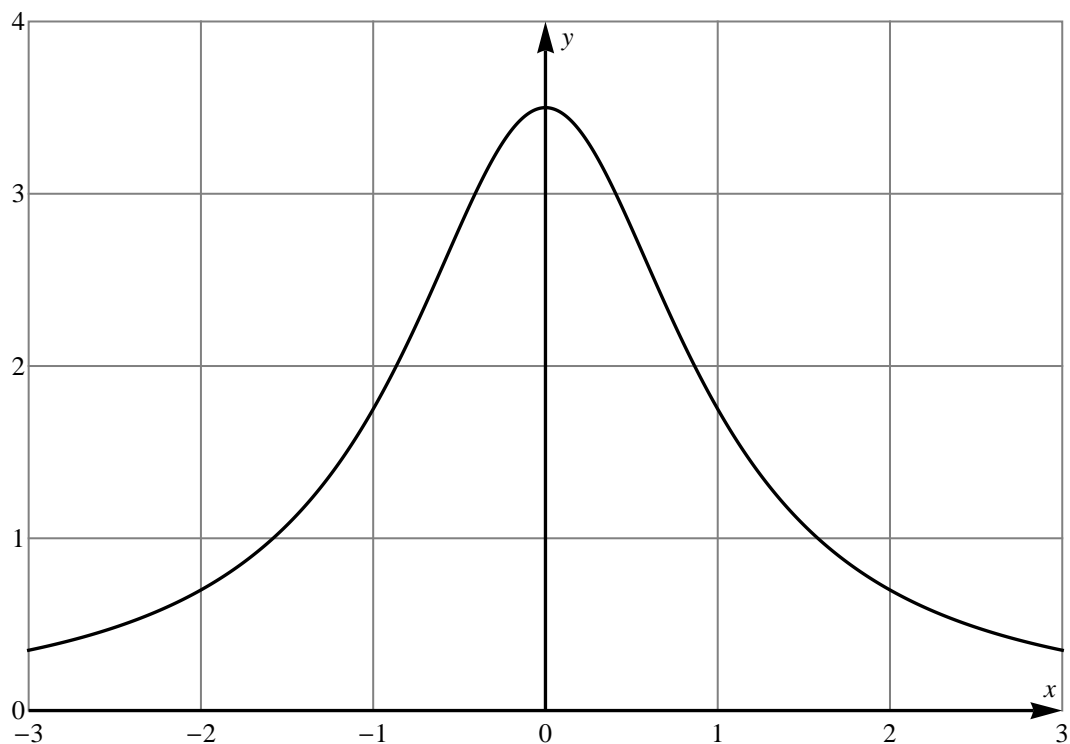


## Homework 3

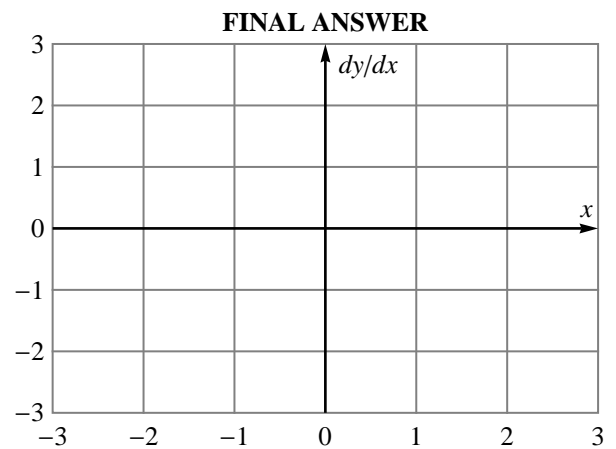
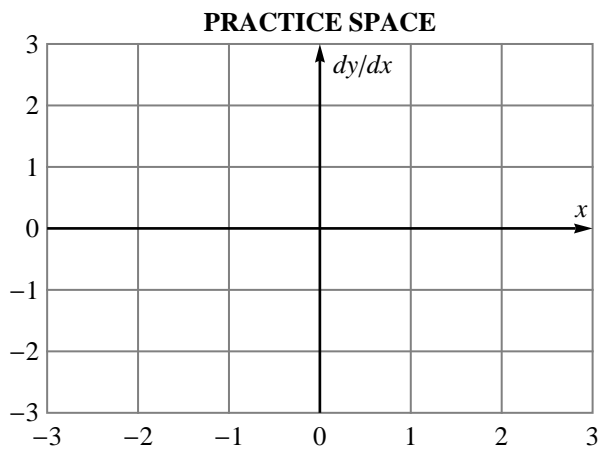
1. (a) The graph of a function  $f(x)$  is shown below. Using a straightedge, draw the tangent lines to this graph at  $x = -3, -2, -1, 0, 1, 2,$  and  $3$ .



- (b) Use your tangent lines to estimate the values of  $f'(x)$  for  $x = -3, -2, -1, 0, 1, 2,$  and  $3$ . Make a table showing your answers

- (c) Given that  $f(x) = \frac{3.5}{1+x^2}$ , use the formula  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to obtain more accurate estimates for the values in part (b). Your answers must be correct to two decimal places.

- (d) Use your answers to part (c) to sketch a rough graph of  $f'(x)$ .



2. Consider the function  $f(x) = 1/x$ .

(a) Use the formula  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to write a limit for  $f'(2)$ .

(b) Use a table of data to evaluate the limit from part (a). Please show the data that you are using.

(c) Use algebra to evaluate the limit from part (a). Please make sure that your work is clear and shows all of your steps. (*Hint: Start by simplifying the fractions, and then cancel an  $h$  from the numerator and denominator.*)

(d) Write a limit for  $f'(x)$  involving both  $x$  and  $h$ .

(e) Use algebra to evaluate the limit from part (d), yielding a general formula for  $f'(x)$ . Please make sure that your work is clear and shows all of your steps.

(f) Is your answer to part (e) consistent with the Power Rule? Explain.

3. Water is draining out of the bottom of a 5000-gallon tank. The volume  $V$  of water (in gallons) remaining in the tank after  $t$  minutes is given by the following formula:

$$V = 5000 \left(1 - \frac{t}{40}\right)^2$$

- (a) Use algebra to find a simplified formula for  $V$  in terms of  $t$ .
- (b) How long does it take for the tank to drain completely? Explain.
- (c) Find the average rate at which water drains from the tank during the first 30 minutes. Make sure your answer includes the proper units.

- (d) Use derivative rules to find a formula for  $\frac{dV}{dt}$  in terms of  $t$ .
- (e) How quickly is water initially draining from the tank? Make sure your answer includes the proper units.
- (f) How quickly is water draining from the tank after 30 minutes? Make sure your answer includes the proper units.