Math 141

Name:_____

Homework 4

1. In the theory of electrical circuits, ***Ohm's law*** describes the relationship between the voltage V across a resistor, the electrical current I passing through the resistor, and a quantity R known as the resistance. The law can be written as follows:

$$V = IR$$

Usually voltage is measured in volts, current is measured in amperes (amps), and resistance is measured in ohms, where 1 ohm $= 1$ volt/amp. In a circuit with variable resistance, the quantities V, I, and R might all depend on time.

(a) Take the derivative of Ohm's law to find an equation involving $\dfrac{dV}{dt}$, $\dfrac{dI}{dt}$, and $\dfrac{dR}{dt}$.

(b) Suppose that the current is increasing at a rate of 0.3 amps per second, while the resistance is holding steady at 4 ohms. How quickly is the voltage across the resistor increasing?

Now suppose that the voltage across the resistor is held constant at 20 volts, while the resistance is steadily increased at a rate of 0.4 ohms per second.

(c) What is the current through the resistor when the resistance reaches 10 ohms?

(d) At what rate is the current changing at that time? Is it increasing or decreasing?

2. A spherical snowball with a radius of 2 inches is melting. The volume V and radius r of the snowball are related by the formula:

$$V = \frac{4}{3}\pi r^3$$

(a) Use the chain rule to find a formula for $\dfrac{dV}{dt}$ in terms of r and $\dfrac{dr}{dt}$.

(b) Because the snowball is melting, the radius of the snowball is decreasing at a rate of 0.7 inches per hour. How quickly is the volume of the snowball decreasing?

3. Boat A is sailing north away from a dock, while boat B is sailing west towards the same dock:
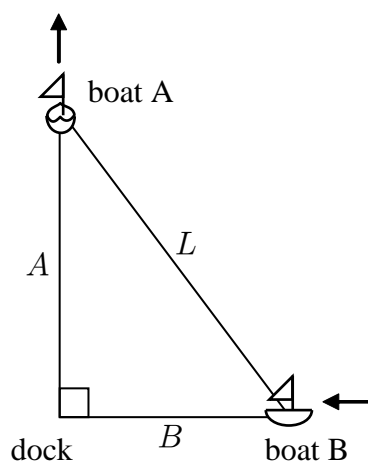
In the picture above, the variable A represents the distance from boat A to the dock, the variable B represents the distance from boat B to the dock, and the variable L represents the distance between the two boats. By the Pythagorean theorem:

$$A^2 + B^2 = L^2$$

(a) Take the derivative of the above equation to find an equation involving $\dfrac{dA}{dt}$, $\dfrac{dB}{dt}$, and $\dfrac{dL}{dt}$.

(b) Is $\dfrac{dA}{dt}$ positive or negative? What about $\dfrac{dB}{dt}$? Explain.

Suppose that boat A is 30 km north of the dock, and is sailing north at a rate of 10 km/ hour. Meanwhile, boat B is 40 km east of the dock, and is sailing west at a rate of 15 km/hour.

(c) What is the present distance between the two boats?

(d) What is $\dfrac{dA}{dt}$? What is $\dfrac{dB}{dt}$?

(e) How quickly is the distance between the boats changing? Is the distance increasing or decreasing?