Math 141

Name: Solutions

Homework 6

1. The human ear is sensitive to an amazing range of sound levels. For example, the sound emitted by a jackhammer is equivalent in magnitude to about 100,000 human voices, while a single human voice is approximately as loud as 100,000 buzzing mosquitoes. In terms of power, a jackhammer emits about 1 watt of sonic energy, a human voice emits about 0.00001 watts, and a buzzing mosquito emits about 0.0000000001 (or 10^{-10}) watts.

   For this reason, the "loudness" of a sound is usually described in terms of ***decibels***, which are a logarithmic unit. The decibel level L of a sound is related to the power P according to the formula

$$L = 10\log_{10}\left(\frac{P}{P_0}\right)$$

   where $P_0 = 10^{-12}$ watts. (The constant P_0 represents the softest sound audible to human ears.)

   (a) Based on the information above, find the decibel levels of the sounds emitted by a jackhammer, a human voice, and a buzzing mosquito.

Jackhammer: $L = 10\log_{10}\left(\frac{1}{10^{-12}}\right) = \boxed{120 \text{ decibels}}$

Human Voice: $L = 10\log_{10}\left(\frac{10^{-5}}{10^{-12}}\right) = \boxed{70 \text{ decibels}}$

Mosquito: $L = 10\log_{10}\left(\frac{10^{-10}}{10^{-12}}\right) = \boxed{20 \text{ decibels}}$

   (b) What is the decibel level of a sound having power P_0?

$L = 10\log_{10}\left(\frac{10^{-12}}{10^{-12}}\right) = \boxed{0 \text{ decibels}}$

(c) Make a table showing the decibel levels for sounds having powers of 0.00001 watts, 0.0001 watts, 0.001 watts, 0.01 watts, 0.1 watts, 1 watt, 10 watts, and 100 watts.

power (watts)	decibels
.00001	70
.0001	80
.001	90
.01	100
.1	110
1	120
10	130
100	140

(d) If the power of a sound is doubled (say, by having two jackhammers instead of one), how much does the decibel level increase?

2 Jackhammers: $10 \log_{10}\left(\frac{2}{10^{-12}}\right) \approx 123.01$ decibels

2 Mosquitos: $10 \log_{10}\left(\frac{2 \cdot 10^{-10}}{10^{-12}}\right) \approx 23.01$ decibels

When the power is doubled, the number of decibels is increased by approximately 3.01.

(e) The band Disaster Area wants to make an awesome 160 decibels of sound at their next rock concert. (They want to be able to brag that they are "as loud as a jet engine".) Unfortunately, no legally available speaker can emit more than 145 decibels of sound. How many speakers will they need?

$160 = 10 \log_{10}\left(\frac{P}{10^{-12}}\right)$

$16 = \log_{10}\left(\frac{P}{10^{-12}}\right)$

$10^{16} = \frac{P}{10^{-12}}$

$P = 10^4 = 10{,}000$ watts

$145 = 10 \log_{10}\left(\frac{P}{10^{-12}}\right)$

$14.5 = \log_{10}\left(\frac{P}{10^{-12}}\right)$

$10^{14.5} = \frac{P}{10^{-12}}$

$P = 10^{2.5} \approx 316.23$ watts

speakers needed $= \frac{10{,}000}{316.23} \approx 31.62$

They need 32 speakers.

2. At the beginning of a biology experiment, a culture of *E. coli* bacteria has a population of 6,000. Ten minutes later, the population has increased to 9,300.

   (a) What was the percentage increase in the population of *E. coli* over the first ten minutes?

   $$\frac{3300}{6000} = 0.55$$

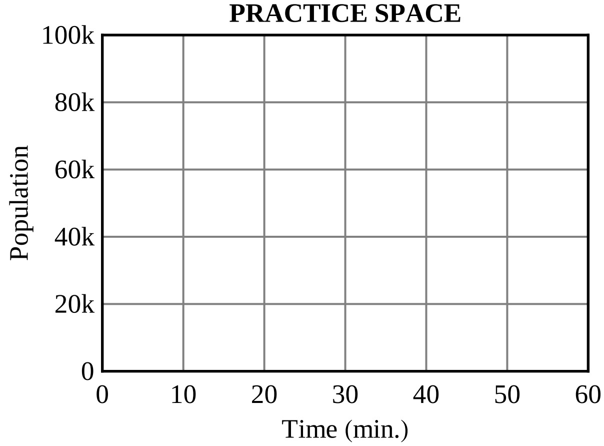
   55%

   (b) Assuming exponential growth, make a table showing the population of *E. coli* after 20 minutes, 30 minutes, 40 minutes, 50 minutes, and 60 minutes.

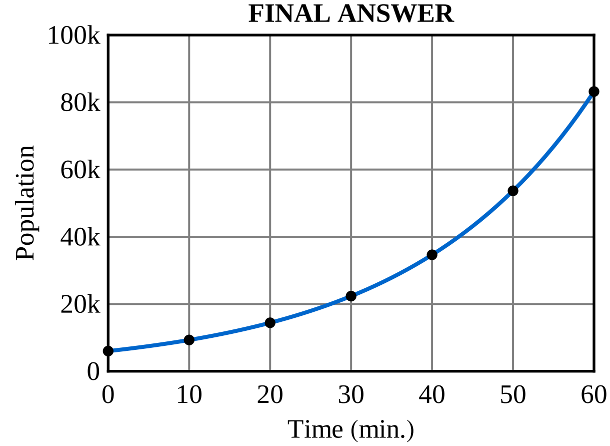
minutes	# bacteria	
0	6000	×1.55
10	9300	×1.55
20	14,415	×1.55
30	22,343	×1.55
40	34,632	×1.55
50	53,679	×1.55
60	83,203	

   (c) Use the following axes to sketch a graph of the *E. coli* population during the first hour of the experiment.

PRACTICE SPACE



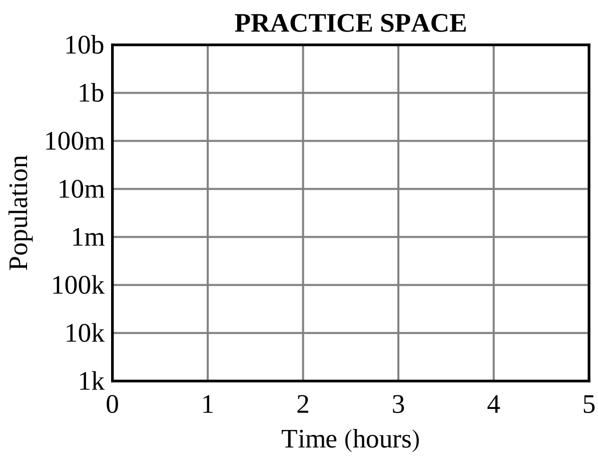
FINAL ANSWER

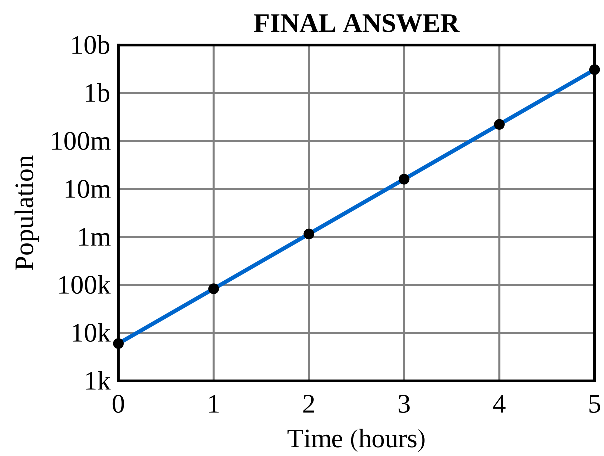


(d) Suppose the bacteria continue to grow exponentially for five hours. Make a table showing the population of *E. coli* at the end of each hour.

hours	# bacteria
0	6000
1	83,203
2	1,153,802
3	16,009,068
4	221,876,858
5	3,076,820,748

(e) Use the following axes to sketch a logarithmic plot of the *E. coli* population over the course of five hours.





(f) Assuming exponential growth, the population P should obey the formula

$$P = b\,e^{kt}$$

where t is the time in minutes, and b and k are constants. What are the values of b and k?

$\boxed{b = 6000}$

$$P = 6000e^{kt}$$

When $t = 10$, $P = 9300$:

$$9300 = 6000e^{10k}$$

$$1.55 = e^{10k}$$

$$\ln(1.55) = 10k$$

$$\boxed{k = \ln(1.55)/10 \approx 0.043825493}$$

(g) The ***generation time*** of a bacterial culture is length of time required for the population to double. What is the generation time of this *E. coli* culture? Your answer must be correct to two decimal places.

$$12{,}000 = 6{,}000e^{.043825493t}$$

$$2 = e^{.043825493t}$$

$$\ln(2) = .043825493t$$

$$t = \frac{\ln(2)}{.043825493} \approx \boxed{15.82 \text{ min.}}$$

(h) Find a formula for $\frac{dP}{dt}$ as a function of t.

$$P = 6000e^{.043825493t}$$

$$\frac{dP}{dt} = 6000e^{.043825493t}(.043825493)$$

$$\boxed{\frac{dP}{dt} = 262.953e^{.043825493t}}$$

(i) Based on your answer to part (h), how quickly was the population increasing at the very beginning of the experiment?

$$t = 0$$

$$\frac{dP}{dt} = 262.953e^{.043825493(0)}$$

$$\frac{dP}{dt} = \boxed{262.953 \text{ bacteria/min.}}$$