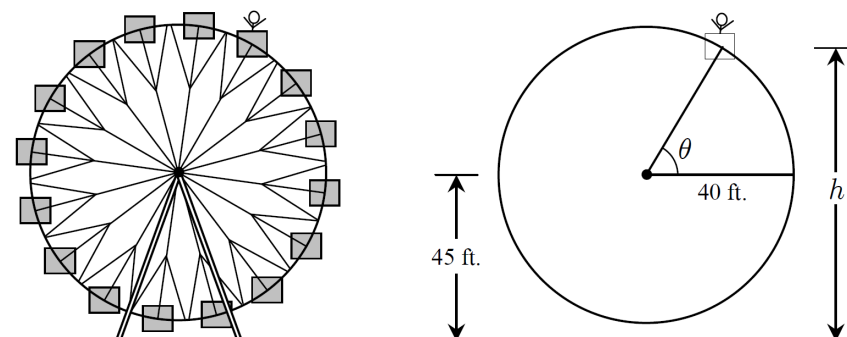


Homework 7

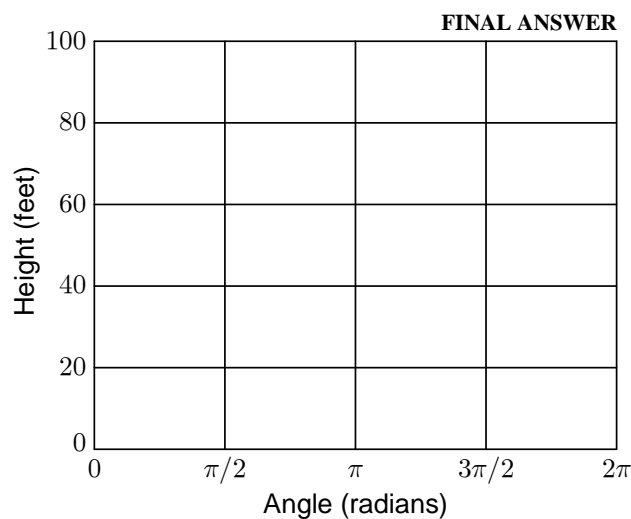
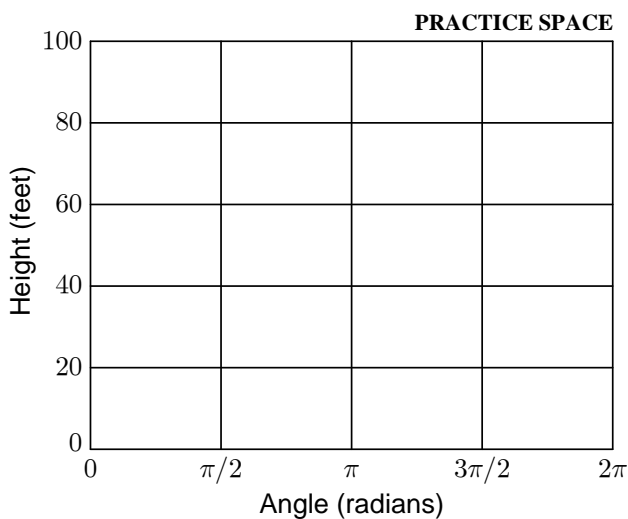
1. Little Joey is riding the Ferris wheel at the county fair:



The wheel has a radius of 40 feet, and the center of the wheel sits 45 feet above the ground. Let h be little Joey's height above the ground, and let θ be the angle shown in the picture above.

(a) Determine little Joey's height h above the ground for $\theta = 0^\circ, 90^\circ, 180^\circ,$ and 270° .

(b) Sketch a graph of h as a function of θ .



(c) Find a formula for h as a function of θ . Make sure that your formula agrees with your answers to parts (a) and (b).

(d) Use your answer to part (c) to find a formula for $\frac{dh}{dt}$ in terms of θ and $\frac{d\theta}{dt}$.

(e) Given that the Ferris wheel is rotating once every 20 seconds, find the value of $\frac{d\theta}{dt}$. Express your answer in radians per second.

(f) How quickly is little Joey ascending when $\theta = 0$?

2. As part of a chemistry experiment, 0.250 moles of butyl chloride ($\text{C}_4\text{H}_9\text{Cl}$) are dissolved in water. The butyl chloride reacts with the water, producing butyl alcohol and hydrochloric acid. Initially, this reaction consumes butyl chloride at a rate of 0.030 moles/min.

Let t be the time in minutes, and let n be the number of moles of butyl chloride remaining.

(a) Assuming that n decays exponentially, find a formula for n in terms of t .

(b) How much butyl chloride will remain after 10 minutes?

(c) How quickly is the butyl chloride being consumed at this time?

(d) How long will it take for 95% of the butyl chloride to be consumed?

3. In astronomy, the *apparent magnitude* is a logarithmic measure of the brightness of a star as seen by an observer on Earth. The apparent magnitude M of a star is related to the brightness B of the observed light by the formula

$$M = -1.09 \ln\left(\frac{B}{B_0}\right)$$

where $B_0 = 2.13 \times 10^{-6}$ lux. Note that brighter stars have *smaller* apparent magnitudes.

- (a) As seen from Earth, the star Polaris (the North Star) has a brightness of 3.5×10^{-7} lux. What is the apparent magnitude of Polaris?

- (b) The brightest star in the night sky is Sirius, with an apparent magnitude of -1.47 . What is the brightness of the light that the Earth receives from this star?

- (c) To an observer on Earth, the sun is approximately 12 billion times as bright as Sirius. What is the apparent magnitude of the sun?

(d) Find a formula relating $\frac{dM}{dt}$ and $\frac{dB}{dt}$.

(e) The apparent magnitude of the variable star Delta Cephei oscillates regularly over the course of several days. At a certain time, the apparent magnitude of Delta Cephei is 3.90, and the magnitude is decreasing at a rate of 0.010/hour. How quickly is the brightness of the star increasing?