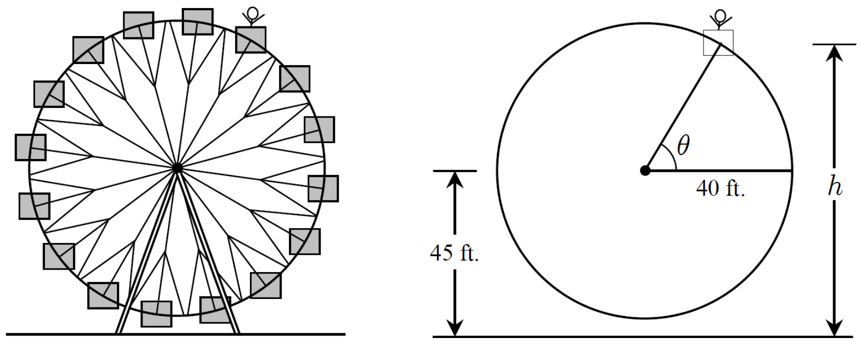
Math 141 Name: Solutions

Homework 7

1. Little Joey is riding the Ferris wheel at the county fair:

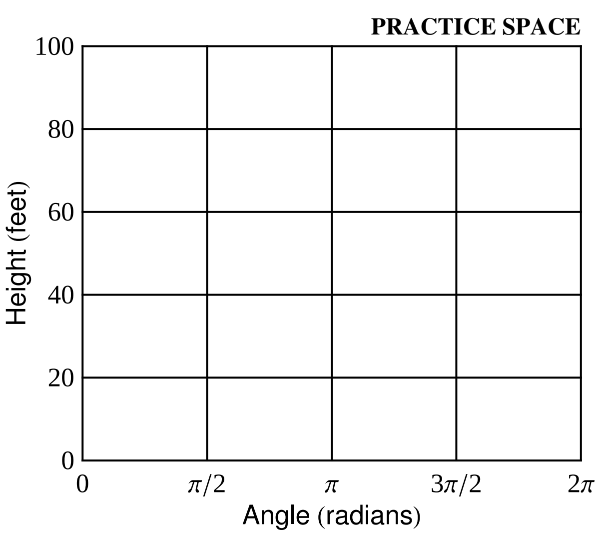


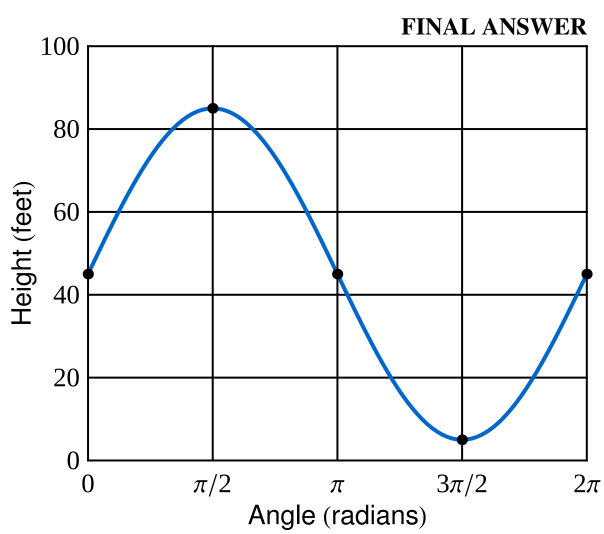
The wheel has a radius of 40 feet, and the center of the wheel sits 45 feet above the ground. Let h be little Joey's height above the ground, and let θ be the angle shown in the picture above.

(a) Determine little Joey's height h above the ground for $\theta = 0°$, $90°$, $180°$, and $270°$.

θ	h
0°	45 ft.
90°	85 ft.
180°	45 ft.
270°	5 ft.

(b) Sketch a graph of h as a function of θ.





(c) Find a formula for h as a function of θ. Make sure that your formula agrees with your answers to parts (a) and (b).

$$h = 40 \sin\theta + 45$$

(d) Use your answer to part (c) to find a formula for $\frac{dh}{dt}$ in terms of θ and $\frac{d\theta}{dt}$.

$$\frac{dh}{dt} = 40 \cos\theta \frac{d\theta}{dt}$$

(e) Given that the Ferris wheel is rotating once every 20 seconds, find the value of $\frac{d\theta}{dt}$. Express your answer in radians per second.

$$\frac{d\theta}{dt} = \frac{2\pi \text{ radians}}{20 \text{ seconds}} = \boxed{\frac{\pi}{10} \text{ rad/sec.}}$$

(f) How quickly is little Joey ascending when $\theta = 0$?

$$\frac{dh}{dt} = 40 \cos(0)\left(\frac{\pi}{10}\right)$$

$$\frac{dh}{dt} = 4\pi \text{ ft./sec.} \approx \boxed{12.57 \text{ ft./sec.}}$$

2. As part of a chemistry experiment, 0.250 moles of butyl chloride (C_4H_9Cl) are dissolved in water. The butyl chloride reacts with the water, producing butyl alcohol and hydrochloric acid. Initially, this reaction consumes butyl chloride at a rate of 0.030 moles/min.

   Let t be the time in minutes, and let n be the number of moles of butyl chloride remaining.

   (a) Assuming that n decays exponentially, find a formula for n in terms of t.

$n = .25e^{kt}$

$\frac{dn}{dt} = .25e^{kt}(k)$

When $t=0$:

$-.03 = .25e^{k(0)}(k)$

$-.03 = .25k$

$k = -.12$

$\boxed{n = .25e^{-.12t}}$

   (b) How much butyl chloride will remain after 10 minutes?

$t = 10$

$n = .25e^{-.12(10)} \approx \boxed{.0753 \text{ moles}}$

   (c) How quickly is the butyl chloride being consumed at this time?

$\frac{dn}{dt} = .25e^{-.12t}(-.12)$

$t=10$: $\frac{dn}{dt} = .25e^{-.12(10)}(-.12) \approx \boxed{-.00904 \text{ moles/min.}}$

   (d) How long will it take for 95% of the butyl chloride to be consumed?

95% consumed ⇒ 5% remaining

$.05(.250) = .0125$ moles remaining

$.0125 = .25e^{-.12t}$

$.05 = e^{-.12t}$

$\ln(.05) = -.12t$

$t = \ln(.05)/-.12 \approx \boxed{24.96 \text{ min.}}$

3. In astronomy, the ***apparent magnitude*** is a logarithmic measure of the brightness of a star as seen by an observer on Earth. The apparent magnitude M of a star is related to the brightness B of the observed light by the formula

$$M = -1.09 \ln\left(\frac{B}{B_0}\right)$$

where $B_0 = 2.13 \times 10^{-6}$ lux. Note that brighter stars have *smaller* apparent magnitudes.

(a) As seen from Earth, the star Polaris (the North Star) has a brightness of 3.5×10^{-7} lux. What is the apparent magnitude of Polaris?

$$M = -1.09 \ln\left(\frac{3.5 \times 10^{-7}}{2.13 \times 10^{-6}}\right) \approx \boxed{1.968}$$

(b) The brightest star in the night sky is Sirius, with an apparent magnitude of -1.47. What is the brightness of the light that the Earth receives from this star?

$$-1.47 = -1.09 \ln\left(\frac{B}{2.13 \times 10^{-6}}\right)$$

$$1.3486 = \ln\left(\frac{B}{2.13 \times 10^{-6}}\right)$$

$$e^{1.3486} = \frac{B}{2.13 \times 10^{-6}}$$

$$B = e^{1.3486}(2.13 \times 10^{-6})$$

$$B \approx \boxed{8.205 \times 10^{-6} \text{ lux}}$$

(c) To an observer on Earth, the sun is approximately 12 billion times as bright as Sirius. What is the apparent magnitude of the sun?

$$M = -1.09 \ln\left(\frac{8.205 \times 10^{-6} \times 12{,}000{,}000{,}000}{2.13 \times 10^{-6}}\right)$$

$$\boxed{M = -26.77}$$

(d) Find a formula relating $\frac{dM}{dt}$ and $\frac{dB}{dt}$.

$$M = -1.09 \ln\left(\frac{B}{B_0}\right)$$

$$\frac{dM}{dt} = -1.09\left(\frac{1}{B/B_0}\right)\left(\frac{1}{B_0}\right)\frac{dB}{dt}$$

$$\frac{dM}{dt} = -1.09\left(\frac{B_0}{B}\right)\left(\frac{1}{B_0}\right)\frac{dB}{dt}$$

$$\boxed{\frac{dM}{dt} = -1.09\left(\frac{1}{B}\right)\frac{dB}{dt}}$$

(e) The apparent magnitude of the variable star Delta Cephei oscillates regularly over the course of several days. At a certain time, the apparent magnitude of Delta Cephei is 3.90, and the magnitude is decreasing at a rate of 0.010/ hour. How quickly is the brightness of the star increasing?

$$3.90 = -1.09 \ln\left(\frac{B}{2.13\times10^{-6}}\right)$$

$$-3.57798 = \ln\left(\frac{B}{2.13\times10^{-6}}\right)$$

$$e^{-3.57798} = \frac{B}{2.13\times10^{-6}}$$

$$B = 5.9495\times10^{-8}$$

$$\frac{dM}{dt} = -1.09\left(\frac{1}{B}\right)\frac{dB}{dt}$$

$$-.01 = -1.09\left(\frac{1}{5.9495\times10^{-8}}\right)\frac{dB}{dt}$$

$$\frac{dB}{dt} = \boxed{5.46\times10^{-10}\ \text{lux/hour}}$$