Exercises: Limits

1–4 ■ Use a table of values to guess the limit.

1. $\displaystyle\lim_{x\to\infty} x^{1/x}$

2. $\displaystyle\lim_{x\to\infty} x - \sqrt{x^2+x}$

3. $\displaystyle\lim_{x\to\infty} \left(1 + \frac{1}{\sqrt{x}}\right)^x$

4. $\displaystyle\lim_{x\to\infty} \sin(x^2)$

5. Use a table of values to estimate the following limit:

$$\lim_{x\to\infty} \left(\frac{x}{x+2}\right)^x$$

Your answer must be correct to four decimal places.

6. Use a table of values to estimate the following limit:

$$\lim_{x\to\infty} \frac{x}{\sqrt{3x^2+1}}$$

Your answer must be correct to four decimal places.

7–14 ■ Identify the largest terms in the numerator and denominator, and use your answers to evaluate the limit.

7. $\displaystyle\lim_{x\to\infty} \frac{x}{1+4x^2}$

8. $\displaystyle\lim_{x\to\infty} \frac{x^3+2}{x+1}$

9. $\displaystyle\lim_{x\to\infty} \frac{6x+1}{2x+5}$

10. $\displaystyle\lim_{x\to\infty} \frac{x^2}{1-x^2}$

11. $\displaystyle\lim_{x\to\infty} \frac{x^2+4x+6}{3x^2+1}$

12. $\displaystyle\lim_{x\to\infty} \frac{1-4x^3}{x^2+2x+1}$

13. $\displaystyle\lim_{x\to\infty} \frac{x^2+3}{2^x}$

14. $\displaystyle\lim_{x\to\infty} \frac{x^4+3^x}{x^5+1}$

15–18 ■ Use a table of values to guess the limit.

15. $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+25}-5}{x}$

16. $\displaystyle\lim_{x\to 0} \frac{4^x-1}{8^x-1}$

17. $\displaystyle\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

18. $\displaystyle\lim_{x\to 1} \frac{x^5-1}{x-1}$

19–22 ■ Evaluate the limit by simplifying the fraction.

19. $\displaystyle\lim_{x\to 0} \frac{(x+5)^2-25}{x}$

20. $\displaystyle\lim_{x\to 0} \frac{(x+1)^3-1}{x}$

21. $\displaystyle\lim_{x\to 3} \frac{x^2-9}{x-3}$

22. $\displaystyle\lim_{x\to 2} \frac{x^2-7x+10}{x-2}$

Answers

1. 1   **2.** -0.5 **3.** infinity   **4.** no limit   **5.** 0.1353   **6.** 0.5774

7. $x/4x^2$; the limit is 0   **8.** x^3/x; the limit is ∞ **9.** $6x/2x$; the limit is 3   **10.** $x^2/-x^2$; the limit is -1

11. $x^2/3x^2$; the limit is $1/3$ **12.** $-4x^3/x^2$; the limit is $-\infty$ **13.** $x^2/2^x$; the limit is 0   **14.** $3^x/x^5$; the limit is ∞

15. 0.1   **16.** $2/3$ **17.** 0.25   **18.** 5   **19.** 10   **20.** 3   **21.** 6   **22.** -3