

Practice Problems: Exam 1

1. Evaluate the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{3x^2 + 5}{5x^2 - 2x}$

(b) $\lim_{x \rightarrow \infty} \frac{1 - x^3}{x^2 + 5x}$

(c) $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{3^x}$

(d) $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$

(e) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

(f) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

2. An oceanographer is taking undersea temperature readings using a thermistor temperature sensor attached to a Niskin bottle. At a depth of 300 meters, she measures a temperature of 21.5 °C. At a depth of 500 meters, she measures a temperature of 14.0 °C.

(a) Use a linear approximation to estimate the temperature at a depth of 600 meters.

(b) Find an approximate formula for the temperature T at a depth of D meters.

(c) Estimate the depth at which the temperature is 16.0 °C

3. The following table shows some values for a function f :

x	$f(x)$
7.1	5.24034184
7.01	5.02040033
7.001	5.00200400
7.0001	5.00020004

Given that $f(7) = 5$, what is $f'(7)$?

4. Let $f(x) = 3^x$. Estimate the value of $f'(1)$. Your answer must be correct to two decimal places.

5. Consider the following limit:

$$\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$$

Use algebra to evaluate this limit. Please make sure that your work is clear and shows all of your steps.

6. In physics, the energy stored in a stretched spring is determined by the equation

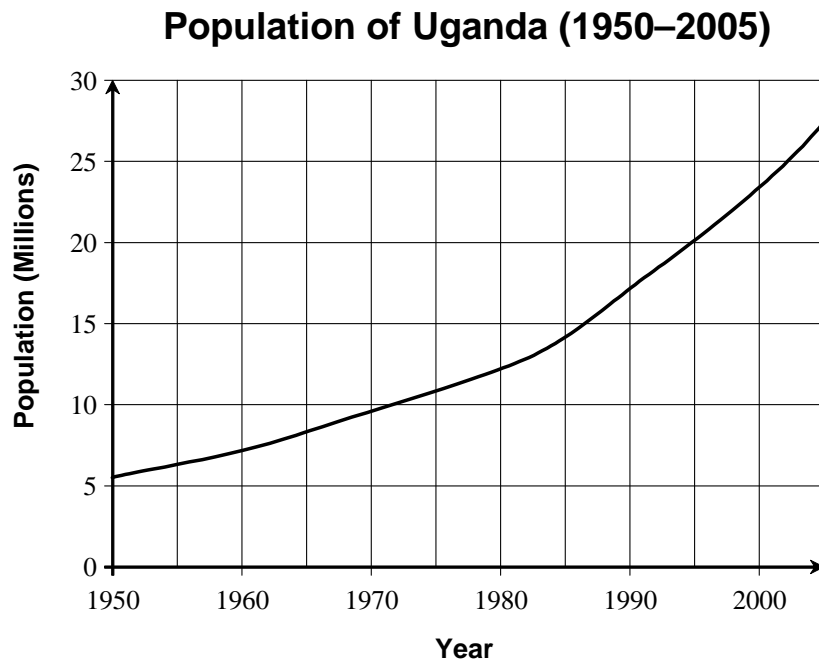
$$E = \frac{1}{2}kx^2,$$

where E is the energy, k is a constant (the *spring constant*), and x represents the distance that the spring has been stretched.

(a) Find a formula for $\frac{dE}{dt}$ in terms of k , x , and $\frac{dx}{dt}$.

(b) A spring with spring constant $k = 0.20$ Joules/cm² is being stretched at a rate of 1.5 cm/sec. How quickly is the energy stored in the spring increasing at the moment that $x = 10$ cm?

7. The following graph shows the population of Uganda since 1950.



(a) Estimate the average annual population growth in Uganda between 1950 and 2005.

(b) Estimate the rate at which Uganda's population was growing in the year 1960.

(c) Estimate how quickly Uganda's population was growing in the year 2005.

(d) Using your answer from part (c), estimate the size of Uganda's population in the year 2015.

8. (a) Find $f'(x)$ if $f(x) = 5x^3 - 3x^2 + 3x - 4$.

(b) Find $\frac{dy}{dx}$ if $y = 5\sqrt{x} + 3$.

(c) Find $\frac{du}{dx}$ if $u = \frac{5x^3 - \sqrt{x}}{x}$.

(d) Find $g'(2)$ if $g(t) = \frac{5}{t^2} - t + 3$.

(e) Find $\frac{dx}{dt}$ if $x = (t^2 + 1)^3$.

(f) Find $\frac{dy}{dx}$ if $y = \frac{1}{(x^3 + x)^5}$.

9. A ball is rolled down an inclined plane. The total distance s rolled by the ball after t seconds is given by the formula

$$s = 5t + 3t^2$$

where s is measured in centimeters.

(a) Find a formula for the velocity of the ball after t seconds.

(b) How long does it take for the velocity to reach 35 cm/s?

10. (a) Find the equation of the tangent line to the curve $y = \sqrt[3]{x}$ at the point $(8, 2)$.

(b) Find $f'(x)$ if $f(x) = x^3(4x + 1)^7$.

(c) Find $\frac{dy}{dx}$ if $y = \frac{x^2}{1 - x^2}$.

(d) Suppose that $f(x) = x^2g(x)$. Given that $g(2) = 4$ and $g'(2) = 3$, find $f'(2)$.

11. In chemistry and physics, *Boyle's Law* describes the relationship between the pressure and volume of a fixed quantity of gas maintained at a constant temperature. The law states that:

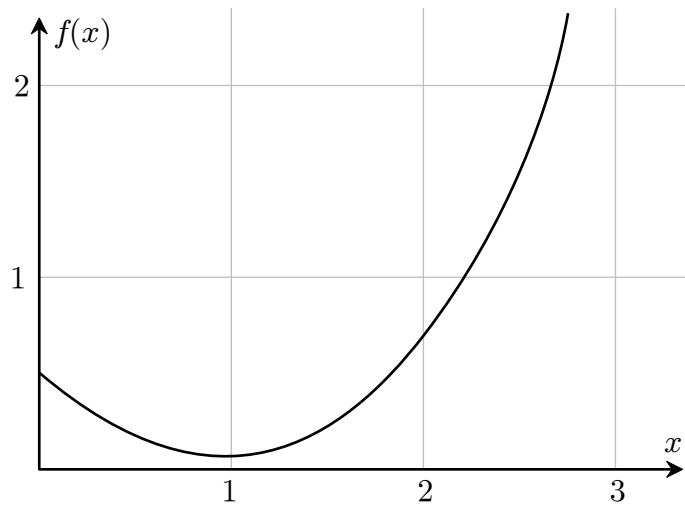
$$PV = \text{a constant}$$

where P is the pressure of the gas, and V is the volume.

(a) Take the derivative of Boyle's law to find an equation relating $\frac{dP}{dt}$, $\frac{dV}{dt}$, P , and V .

(b) A sample of gas is placed in a cylindrical piston. At the beginning of the experiment, the gas occupies a volume of 250 cm³, and has a pressure of 100 kPa. The piston is slowly compressed, decreasing the volume of the gas at a rate of 50 cm³/min. How quickly will the pressure of the gas initially increase?

12. The graph of a function $f(x)$ is shown below:



You know that $f(2) = 0.7$, and $f'(2) = 1.3$.

(a) Use a linear approximation to estimate $f(2.1)$.

(b) Is your answer to part (a) an overestimate or an underestimate? Explain.