# Practice Problems: Exam 2

1. A chemist prepares a 100-mg sample of ^{210}Po. Because of radioactive decay, the mass of the sample after one day is only 99.5 mg.

   (a) Find a formula for the mass of the sample after x days.

   (b) How long will it take for the mass of the sample to reach 50 mg? Round your answer to the nearest day.

2. At noon, a bacteria culture contains 500 bacteria with an instantaneous growth rate of 150 bacteria/hour.

   (a) Assume that the number of bacteria in the culture is increasing exponentially. Find a formula for the number of bacteria after t hours.

   (b) How many bacteria are in the culture at 7:30 pm?

   (c) At what time does the culture contain 15,000 bacteria?

3. A lighthouse sits on a small island near a rocky shoreline, emitting a rotating beam of light. The lighthouse is 3 km from the shore, and it emits a beam of light that rotates four times per minute:
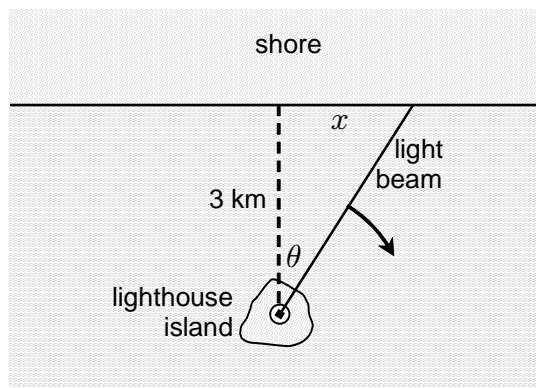
   (a) Find a formula for x in terms of θ.

   (b) Using your answer to part (a), find a formula for $\dfrac{dx}{dt}$ in terms of θ and $\dfrac{d\theta}{dt}$.

   (c) Based on your formula from part (b), how quickly is the end of the light beam moving along the shoreline when $\theta = 30$ degrees?

4. The following picture shows the curve defined by the equation $x^3 + y^3 = 9y$.
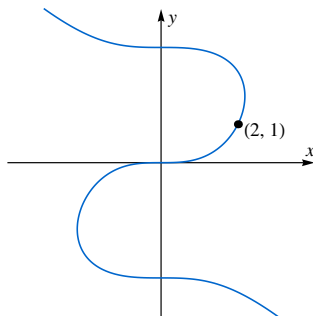
(a) Use implicit differentiation to find a formula for $\dfrac{dy}{dx}$ in terms of x and y.

(b) Use your answer to part (a) to find the slope of the curve at the point $(2,1)$.

5. (a) Find $\dfrac{dy}{dx}$ if $y = \dfrac{3x - 5}{x^2 - 4}$.

(b) Find $f'(x)$ if $f(x) = (\ln x - \ln 2)^2$.

(c) Find $\dfrac{d}{dx}\left(x^2 e^{-3x}\right)$.

(d) Find $\dfrac{dy}{dx}$ if $y = x\sec(x^2 + 1)$.

(e) Find $g'(u)$ if $g(u) = \sin^{-1}(3x^2)$.

(f) Find $\dfrac{dy}{dx}$ if $y = \tan^3(x^2 + 1)$.

(g) Find $\dfrac{d\theta}{dt}$ if $\theta = \arctan(\ln t)$.

6. The following table shows some information about a function:

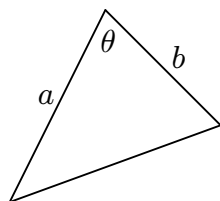| x	0.0	0.2	0.4	0.6	0.8	1.0
$f(x)$	1.2	1.8	2.8	4.2	6.0	8.2

Based on this data, estimate each of the following quantities as accurately as you can.

(a) $f'(0.5)$

(b) $f'(0.3)$

(c) $f''(0.4)$

7. The following formula lets you determine the area of a triangle given the lengths of two sides and the measure of the angle between them:

$$A = \frac{1}{2}ab\sin\theta$$

For this problem, the lengths a and b are both constant, with $a = 4$ inches and $b = 5$ inches.

(a) Suppose that θ is increasing at a rate of 0.15 rad/min. How quickly is the area of the triangle increasing at the instant that $\theta = 0.4$ rad?

(b) Suppose instead that the area is increasing at a rate of 0.5 in^2/min. How quickly is θ increasing at the instant that the area is 6 square inches?

8. (a) Find $f''(x)$ if $f(x) = x\cos x$.

(b) Find $\dfrac{d^2y}{dx^2}$ if $y = \sqrt{3x+1}$.

(c) Find $\dfrac{dy}{dx}$ if $y^3 + \sin y = x + \cos x$.

9. Consider the function $f(x) = \dfrac{36}{x} + 25x$.

(a) Find the absolute minimum value of $f(x)$ on the interval $[0.5, 2.5]$.

(b) Find the absolute maximum value of $f(x)$ on the interval $[0.5, 2.5]$.

10. Sketch the graph of the function f, given the following information.

General Information	**First Derivative**	**Second Derivative**
$f(-1) = 3$ and $f(1) = 0$.	$f'(-1) = 0$ and $f'(1) = 0$.	$f''(0) = 0$ and $f''(2) = 0$.
f is continuous.	$f'(x) > 0$ for $x < -1$.	$f''(x) < 0$ for $x < 0$.
$\lim\limits_{x\to-\infty} f(x) = -\infty$.	$f'(x) < 0$ for $-1 < x < 1$.	$f''(x) > 0$ for $0 < x < 2$.
$\lim\limits_{x\to\infty} f(x) = 4$.	$f'(x) > 0$ for $x > 1$.	$f''(x) < 0$ for $x > 2$.

11. The function $f(x)$ is graphed below:
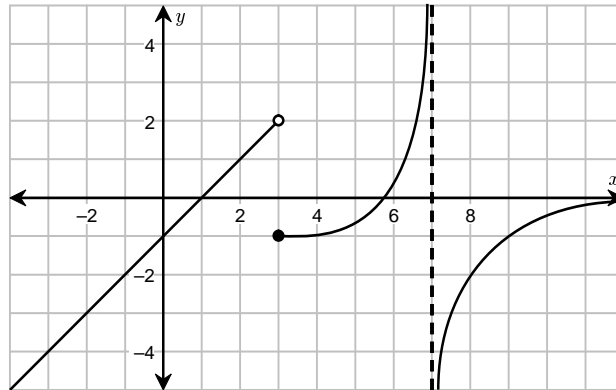
(a) What is $\lim\limits_{x \to 3^-} f(x)$?

(b) What is $\lim\limits_{x \to \infty} f(x)$?

(c) What is $\lim\limits_{x \to 7^+} f(x)$?

(d) For what values of x is $f(x)$ not continuous?

(e) Is $f''(8)$ positive or negative?

(f) What is $f''(-2)$?

12. Consider the following piecewise-defined function:

$$f(x) = \begin{cases} 3x+1 & \text{if} \quad x < -3 \\ 1-x^2 & \text{if} \quad -3 \le x \le -1 \\ 1/x & \text{if} \quad -1 < x \le 1 \\ 5-x & \text{if} \quad x > 1 \end{cases}$$

(a) What is $f(-3)$?

(b) What is $\lim\limits_{x \to -1^+} f(x)$?

(c) What is $\lim\limits_{x \to -1^-} f(x)$?

(d) What is $\lim\limits_{x \to -3} f(x)$?

(e) What is $\lim\limits_{x \to 1} f(x)$?

(f) For what values of x is $f(x)$ not continuous?

13. Evaluate the following limits:

(a) $\lim\limits_{x \to 3^+} \dfrac{x-5}{x^2-9}$

(b) $\lim\limits_{x \to 2^+} \ln(x-2)$

(c) $\lim\limits_{x \to -5} \dfrac{|x+5|}{x+5}$

(d) $\lim\limits_{x \to 0^-} \dfrac{x}{\sqrt{4x^2-x^4}}$

14. A penny is dropped from the top of a 400-foot-tall building. The penny's initial velocity is 0, but it accelerates downwards due to gravity at a rate of 32 feet/second^2. How long does it take for the penny to hit the ground?

15. Carol plans to create a large open box (a box without a top) from a piece of cardboard. She has a cardboard rectangle with side lengths 6 feet and 12 feet. She will cut off a square with side length a from each corner, and then fold the resulting flaps up to create a box.
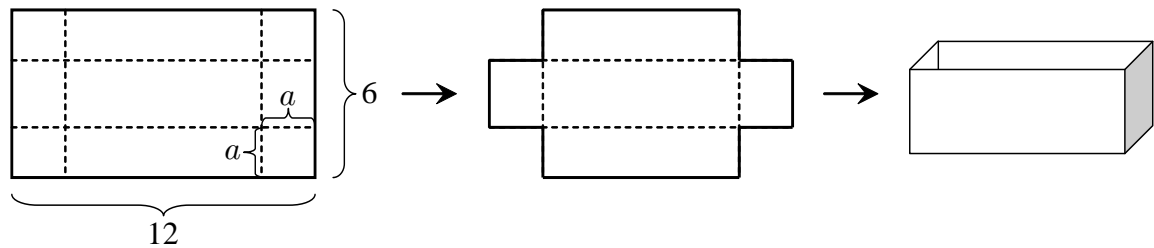
Determine the maximum possible volume of the box.