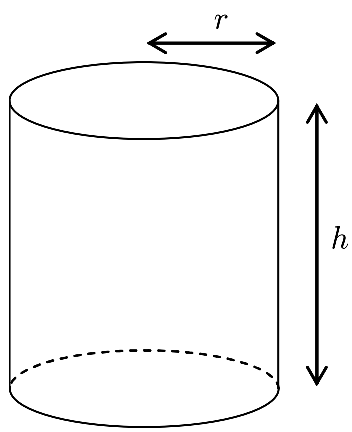
Practice Problems: Final Exam

1. Estimate the integral $\displaystyle\int_0^1 \frac{1}{1+x^3}\,dx$ using five rectangles and left endpoints.

2. A cylindrical cup with an open top must have a volume of 300 cm^3:



   Find the dimensions of the cup that minimize the amount of material used.

3. Evaluate the following integrals.

   (a) $\displaystyle\int_1^2 x(x+2)\,dx$

   (b) $\displaystyle\int_1^4 \sqrt{x}\,dx$

   (c) $\displaystyle\int_0^2 \frac{1}{5x+1}\,dx$

   (d) $\displaystyle\int_0^{\pi/2} \cos(3x)\,dx$

   (e) $\displaystyle\int_{-1}^1 (1-x)^5\,dx$

   (f) $\displaystyle\int_0^{\pi/4} \sec^2 x\,dx$

   (g) $\displaystyle\int_0^2 (e^x+1)^2\,dx$

   (h) $\displaystyle\int_0^1 \left(x^3\cos x + 3x^2\sin x\right)dx$

   (i) $\displaystyle\int_0^4 2x\sqrt{x^2+9}\,dx$

   (j) $\displaystyle\int_0^1 \frac{e^{2x}}{1+e^{2x}}\,dx$

4. The following table shows the velocity of an object over the course of one second.

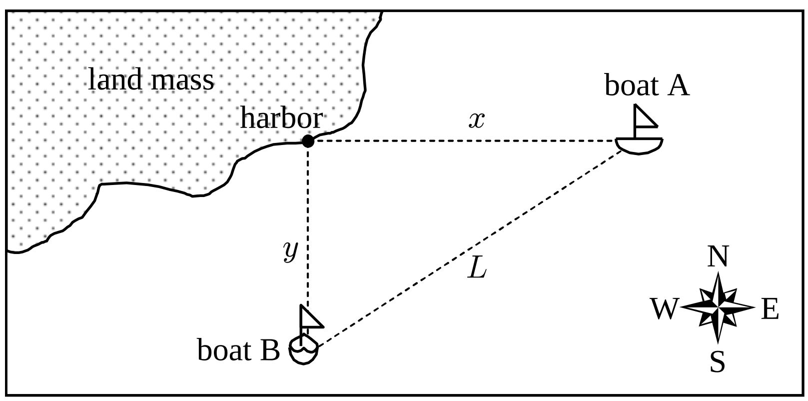
t (seconds)	0.0	0.2	0.4	0.6	0.8	1.0
v (meters/sec)	3.6	4.0	4.3	4.6	4.8	4.9

(a) Estimate the distance traveled by the object between $t = 0$ and $t = 1$.

(b) Estimate the acceleration of the object at $t = 0.4$.

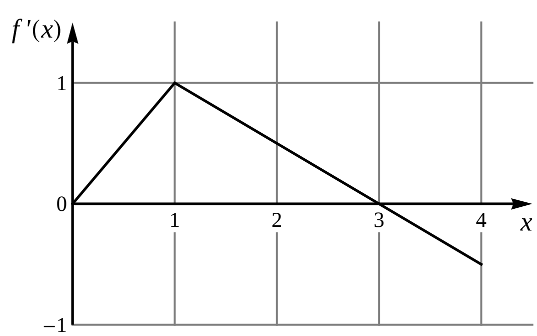
(c) Estimate the velocity of the object at $t = 0.43$.

5. Two boats are both sailing away from a harbor, as shown in the picture below.



Boat A is 80 miles east of the harbor, and is sailing due east at 15 miles/hour. Boat B is 60 miles south of the harbor, and is sailing due south at 20 miles/hour. How quickly is the distance between the two boats increasing?

6. The following graph shows the **derivative** of a function $f(x)$:

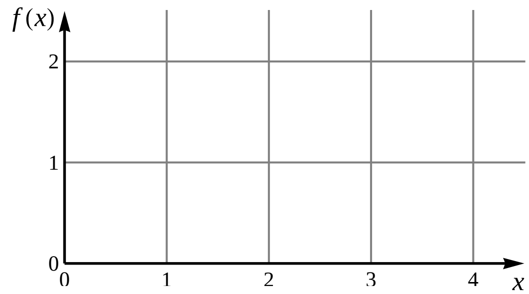


(a) Given that $f(0) = 1/2$, evaluate $f(1)$ and $f(4)$.

(b) Find the maximum value of $f(x)$ on the interval $[0,4]$.

(c) For what values of x is $f''(x)$ positive? For what values of x is $f''(x)$ negative?

(d) Sketch a graph of the function $f(x)$:



7. (a) Find $g'(u)$ if $g(u) = \sqrt{1+u^2}$.

(b) Find $\dfrac{dx}{d\theta}$ if $x = \dfrac{1}{5}\sec\left(\dfrac{\theta}{2}\right)$

(c) Find $\dfrac{dy}{dx}$ if $y = \ln(\sin x)$.

(d) Find $f'(x)$ if $f(x) = \displaystyle\int_3^x \frac{1}{1+u^4}\,du$.

(e) Find $\dfrac{dy}{dx}$ if $4x^2 + y^2 = 9$.

(f) Find $f(x)$ if $f'(x) = x^2 + e^{2x}$ and $f(0) = 4$.

8. Consider the following piecewise-defined function:

$$f(x) = \begin{cases} 3x-1 & \text{if} \quad x \le -2 \\ x^2-2 & \text{if} \quad -2 < x \le 1 \\ 2x-3 & \text{if} \quad 1 < x \le 3 \\ 6-x & \text{if} \quad x > 3 \end{cases}$$

(a) What is $f(-2)$?

(b) What is $\displaystyle\lim_{x\to -2^+} f(x)$?

(c) What is $\displaystyle\lim_{x\to -2^-} f(x)$?

(d) What is $\displaystyle\int_1^6 f(x)\,dx$?

(e) For what values of x is $f(x)$ not continuous?

(f) For what values of x is $f(x)$ not differentiable?

9. Evaluate the following limits:

   (a) $\lim_{x\to\infty} \frac{x}{x+5}$

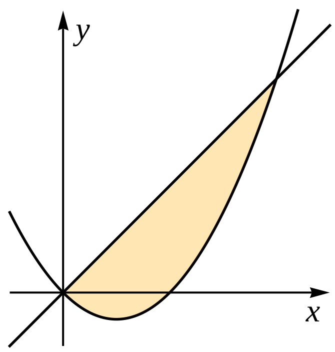
   (b) $\lim_{x\to 1^+} \frac{x}{1-x}$

   (c) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

   (d) $\lim_{x\to \pi/2^+} \tan x$

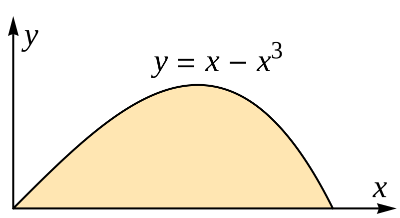
   (e) $\lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx$

10. The following figure shows the region between the curves $y = x^2 - x$ and $y = x$.



    Find the area of this region.

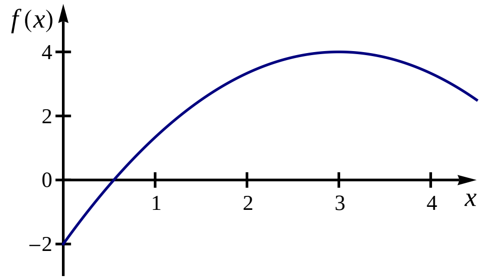
11. The following figure shows the region in the first quadrant lying under the graph of $y = x - x^3$.



    (a) Find the area of this region.

    (b) Find the volume of the solid obtained by rotating this region around the x-axis.

12. The function $f(x)$ is graphed below.



Suppose that $f(3) = 4$ and $\int_0^3 f(x)\,dx = 6$.

(a) Estimate the value of $\int_0^{3.02} f(x)\,dx$.

(b) Estimate the value of b for which $\int_0^b f(x)\,dx = 6.2$.

13. In physics, the kinetic energy of a moving object is given by the formula

$$K = \frac{1}{2}mv^2$$

where m is the mass and v is the velocity. Suppose an object with a mass of 2.00 kg is accelerating at a rate of $6.00\ \mathrm{m/s^2}$. How quickly is the kinetic energy of the object increasing when the velocity is $23.0\ \mathrm{m/s}$?

14. Use differentials to estimate the area of the shaded region in the following figure.