Exercises: Related Rates

1–12 ■ Use the given equation to find a formula for $\frac{dP}{dt}$ in terms of Q, R, $\frac{dQ}{dt}$, and $\frac{dR}{dt}$.

1. $P = 6Q$

2. $P = Q + 2R + 3$

3. $P = QR$

4. $P = 5QR - Q$

5. $P = Q^3$

6. $P = 5\sqrt{R}$

7. $P = Q^2 + R^2$

8. $P = Q + \frac{5}{R}$

9. $P = QR^3$

10. $P = Q^2\sqrt{R}$

11. $P = \frac{Q}{R}$

12. $P = \frac{\sqrt{Q}}{R^3}$

13. The radius of a cylinder is constant at 5 inches, while the height is increasing at a rate of 2 inches/sec. How quickly is the volume of the cylinder increasing?

14. The base of a triangle is constant at 4 cm, while the height is increasing at a rate of 0.3 cm/sec. How quickly is the area of the triangle increasing?

15. The length of a rectangle is increasing at a rate of 5 feet/min, while the width is decreasing at a rate of 3 feet/min. How quickly is the area of the rectangle changing when the length is 20 feet and the width is 10 feet? Is the area increasing or decreasing?

16. The magnetic flux Φ through a loop of wire depends on the magnetic field B and the area A according to the formula

$$\Phi = AB.$$

(a) Suppose that the area of a loop is constant at 10 cm^2, while the magnetic field is increasing at a rate of 0.30 Tesla/sec. How quickly is the flux through the loop increasing?

(b) Suppose instead that the area is increasing at a rate of 2.0 cm^2/sec, while the magnetic field is increasing at a rate of 0.15 Tesla/sec. How quickly is the flux increasing when the area is 10 cm^2 and the magnetic field is 0.80 Tesla?

17. The side length of a square is increasing at a rate of 3 cm/sec. How quickly is the area of the square increasing when the side length is 20 cm?

18. The radius of a circle is increasing at a rate of 5 cm/min. How quickly is the area of the circle increasing when the radius is 30 cm?

19. The radius of a sphere is increasing at a rate of 1.2 cm/hour.

(a) How quickly is the volume of the sphere increasing when the radius is 10 cm?

(b) How quickly is the surface area of the sphere increasing at that time?

20. The radius of a cylinder is increasing at a rate of 2.0 cm/min, while the height is increasing at a rate of 1.5 cm/min. How quickly is the volume of the cylinder increasing when the radius is 12 cm and the height is 10 cm?

Answers

1. $6\dfrac{dQ}{dt}$ **2.** $\dfrac{dQ}{dt}+2\dfrac{dR}{dt}$ **3.** $R\dfrac{dQ}{dt}+Q\dfrac{dR}{dt}$ **4.** $5\left(R\dfrac{dQ}{dt}+Q\dfrac{dR}{dt}\right)-\dfrac{dQ}{dt}$ **5.** $3Q^2\dfrac{dQ}{dt}$ **6.** $\dfrac{5}{2}R^{-1/2}\dfrac{dR}{dt}$

7. $2Q\dfrac{dQ}{dt}+2R\dfrac{dR}{dt}$ **8.** $\dfrac{dQ}{dt}-5R^{-2}\dfrac{dR}{dt}$ **9.** $R^3\dfrac{dQ}{dt}+3QR^2\dfrac{dR}{dt}$ **10.** $2QR^{1/2}\dfrac{dQ}{dt}+\dfrac{1}{2}Q^2R^{-1/2}\dfrac{dR}{dt}$

11. $R^{-1}\dfrac{dQ}{dt}-QR^{-2}\dfrac{dR}{dt}$ **12.** $\dfrac{1}{2}Q^{-1/2}R^{-3}\dfrac{dQ}{dt}-3Q^{1/2}R^{-4}\dfrac{dR}{dt}$ **13.** 50π inches^3/sec **14.** 0.6 cm^2/sec

15. decreasing at 10 feet^3/min **16.** (a) 3.0 Tesla$\cdot$cm^2/sec (b) 3.1 Tesla$\cdot$cm^2/sec **17.** 120 cm^2/sec **18.** 300π cm^2/min

19. (a) 480π cm^3/hour (b) 96π cm^2/hour **20.** 696π cm^3/min