Math 261 — Insert Name Here

Spring 2013 — Prof. Belk

Sample Assignment 2

Exercise 1.5.11

Part (1)

(1) $(\forall x \text{ in } U)[R(x) \to C(x)]$	
(2) $(\forall x \text{ in } U)[T(x) \to R(x)]$	
Consider an arbitrary a in U.	
(3) $R(a) \to C(a)$	(1), Universal Instantiation
(4) $T(a) \to R(a)$	(2), Universal Instantiation
(5) $T(a) \to C(a)$	(3), (4), Hypothetical Syllogism
(6) $\neg C(a) \to \neg T(a)$	(5), Contrapositive
(7) $(\forall x \text{ in } U)[\neg C(x) \to \neg T(x)]$	(6), Universal Generalization

Part (2)

(1) $(\forall a \text{ in } V)[N(a) \to B(a)]$	
(2) $(\exists b \text{ in } V)[N(b) \wedge D(b)]$	
(3) $N(p) \wedge D(p)$	(2), Existential Instantiation
(4) $N(p)$	(3), Simplification
(5) $N(p) \to B(p)$	(1), Universal Instantiation
(6) $B(p)$	(4), (5), Modus Ponens
(7) $D(p)$	(3), Simplification
(8) $B(p) \wedge D(p)$	(6), (7), Adjunction
(9) $(\exists c)[B(c) \wedge D(c)]$	(8), Existential Generalization

Exercise 1.4.1

Part (1)

From the first premise, we can use Simplification and Addition to deduce $P \vee Q$. Combining this with the second premise yields R.

Part (3)

The contrapositive of the second premise is $F \to G$, and combining this with the first premise gives $E \to G$. We now have $E \vee H$, $E \to G$, and $H \to I$, so we can use Constructive Dilemma to conclude $G \vee I$.

Some Proofs

The following are all valid proofs of theorem 2.2.2:

Theorem. *Let a, b, and c be integers. If $a|b$ and $b|c$, then $a|c$.*

Proof. Suppose that $a|b$ and $b|c$. Hence there are integers q and r such that $aq = b$ and $br = c$. Define an integer k by $k = qr$. Then $ak = a(qr) = (aq)r = br = c$. Because $ak = c$, it follows that $a|c$. □

Theorem. *Let a, b, and c be integers. If $a|b$ and $b|c$, then $a|c$.*

Proof. Suppose that $a|b$ and $b|c$. By the definition of divides, there exist integers s and t for which $as = b$ and $bt = c$. Let $u = st$. Then u is an integer, and $au = ast = bt = c$, which proves that $a|c$. □

Theorem. *Let a, b, and c be integers. If $a|b$ and $b|c$, then $a|c$.*

Proof. Assuming that $a|b$ and $b|c$, there must be integers m and n so that $am = b$ and $bn = c$. Then mn is an integer and $a(mn) = (am)n = bn = c$, and therefore $a|c$. □

Here is a formal two-column proof of the same theorem:

(1) $a\|b$	
(2) $b\|c$	
(3) $(\exists x)(ax = b)$	(1), Definition of Divides
(4) $(\exists x)(bx = c)$	(2), Definition of Divides
(5) $aq = b$	(3), Existential Instantiation
(6) $br = c$	(4), Existential Instantiation
(7) $a(qr) = (aq)r$	Associative Law
(8) $a(qr) = br$	(5), (7), Substitution
(9) $a(qr) = c$	(6), (8), Substitution
(10) $(\exists x)(ax = c)$	(9), Existential Generalization
(11) $a\|c$	(10), Definition of Divides