

Math 316 Homework 1

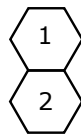
Due Friday, February 10

Solutions must be written in \LaTeX . You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (a) Consider 5-pile Nim with piles of size 10, 22, 28, 40, and 50. Which player has a winning strategy? If player 1 has a winning strategy, what should their first move be?
(b) Consider 5-pile Nim with piles of size 23, 39, 45, 17, and 12. Which player has a winning strategy? If player 1 has a winning strategy, what should their first move be?

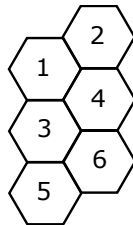
2. Consider a modified version of Nim called Poison. The game Poison is played exactly as Nim, except that the player to take the last object is the loser (and the other player is the winner).
(a) Does one of the players in Poison have a winning strategy?
(b) Explain how to determine which player has a winning strategy in Poison (given the initial number of piles and objects in each pile). Describe that player's winning strategy?

3. In this problem, we will consider games of Hex where the length of the board is not same as the width. For these problems assume that Player 1 plays first and wins by creating a path from the top to the bottom, and Player 2 plays second and wins by creating a path from left to right. (Note: The Hex boards below all have numbers in the hexagons. The numbers are not part of the game, they are just there to help you describe the winning strategies — for example, you can say that Player 1 should start by taking Hex 5.)
(a) Consider 1×2 Hex where the board looks like:



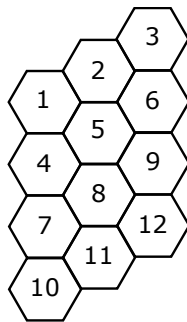
Which player has a winning strategy in this game? Describe a winning strategy for that player.

(b) Consider 2×3 Hex where the board looks like:



Which player has a winning strategy in this game? Describe a winning strategy for that player.

(c) Consider 3×4 Hex where the board looks like:



Which player has a winning strategy in this game? Describe a winning strategy for that player.

4. Read about David Gale's game of Chomp in the textbook (see Section 3.4).

- (a) Which of the two players has a winning strategy in a game of Chomp played on a $2 \times \infty$ board? Justify your answer.
- (b) Which of the two players has a winning strategy in a game of Chomp played on an $m \times \infty$ board, where m is any finite number? Justify your answer.